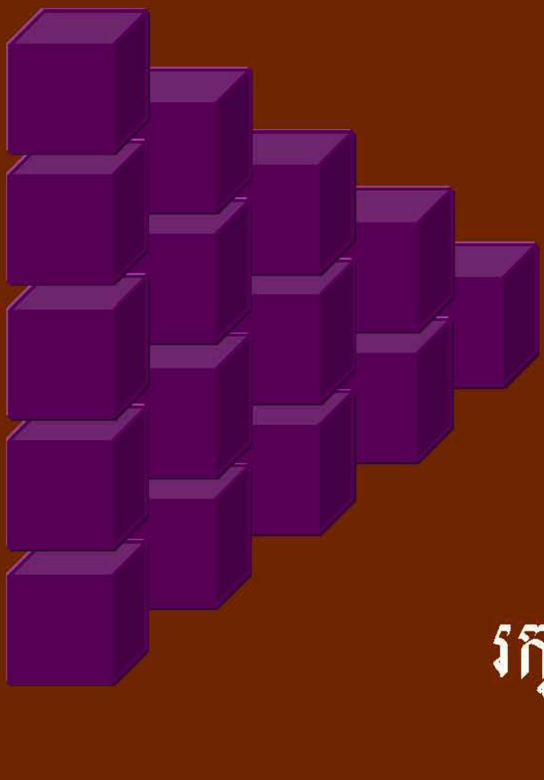
រៀបរៀងដោយ លឹម ផល្គុន និង សែន ពិសិដ្ឋ
បរិញ្ញាបត្រគណិតវិទ្យា និង ពាណិជ្ជកម្ម

គន្លឹះ គណនា លីមីតនៃអនុគមន៍

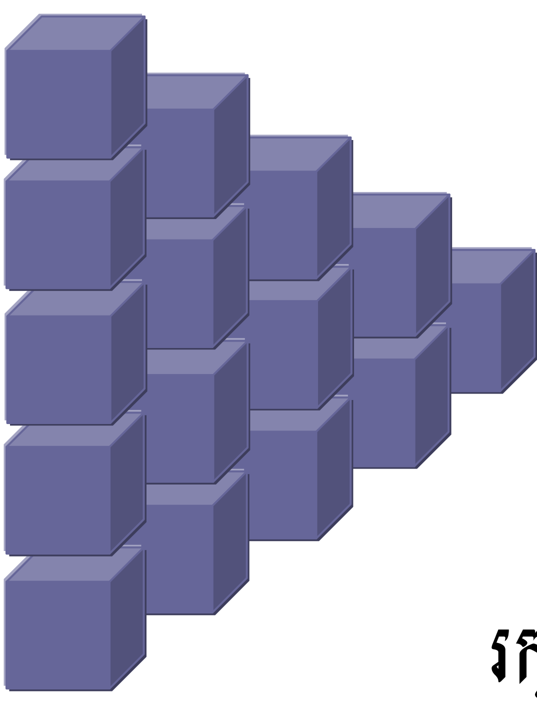
- សង្ខេបរូបមន្ត
- វិធីសាស្ត្រដោះស្រាយ
- លំហាត់គំរូ
- លំហាត់អនុវត្តន៍



រៀបរៀងដោយ លឹម ផល្លុន និង សែន ពិសិដ្ឋ
បរិញ្ញាបត្រ័គណិតវិទ្យា និង ពាណិជ្ជកម្ម

គន្លឹះ គណនា លីមីតនៃអនុគមន៍

- សង្ខេបរូបមន្ត
- វិធីសាស្ត្រដោះស្រាយ
- លំហាត់គំរូ
- លំហាត់អនុវត្តន៍



អ្នកចូលរួមត្រួតពិនិត្យបច្ចេកទេស

លោក លឹម ឆុន

លោក សែន ពិសិដ្ឋ

លោកស្រី ឆុយ វិណោ

លោក ទិត្យ ម៉េង

លោក ព្រឹម សុនិត្យ

លោក ផល ប៊ុនឆាយ

អ្នកត្រួតពិនិត្យអក្ខរាវិរុទ្ធ

លោក លឹម មិគ្គសិរ

ការីកុំព្យូទ័រ

កញ្ញា លី គុណ្ណាកា

អ្នកនិពន្ធ និង រៀបរៀង

លោក លឹម ផល្លុន និង លោក សែន ពិសិដ្ឋ

អារម្ភកថា

សៀវភៅ **គន្លឹះគណនាលីមីត** ដែលអ្នកសិក្សាកំពុងកាន់នៅក្នុងដៃនេះ ខ្ញុំបាទបានរៀបរៀងឡើងក្នុងគោលបំនងទុកជាឯកសា ជំនួយស្មារតីដល់អ្នកសិក្សាទាំងអស់ដែល ចង់ចេះ ចង់ដឹងអំពីមេរៀនលីមីតនេះ ។

នៅក្នុងសៀវភៅនេះយើងខ្ញុំបានសរសេរវិធីសាស្ត្រសម្រាប់ដោះស្រាយ និង អមជាមួយលំហាត់គំរូ ដែលអាចឱ្យអ្នកសិក្សា ងាយយល់ និង ឆាប់ចងចាំអំពីវិធីសាស្ត្រដោះស្រាយនីមួយៗ ។

យើងខ្ញុំសង្ឃឹមថា សៀវភៅ **គន្លឹះគណនាលីមីត** មួយក្បាលនេះនឹងអាចចូលរួមចំណែកក្នុងការពង្រីកចំណេះដឹងរបស់អ្នកសិក្សាជាពុំខានឡើយ ។

យើងខ្ញុំរង់ចាំជានិច្ច នូវមតិរិះគន់ពីអ្នកសិក្សាក្នុងគ្រប់មជ្ឍដ្ឋាន ដោយក្តីសោមនស្សរីករាយ បំផុត ដើម្បីកែលំអសៀវភៅនេះ ឱ្យកាន់តែមានសុក្រិត្យភាពថែមទៀត ។

សូមជូនពរដល់អ្នកសិក្សាទាំងអស់ជួបតែសំណាងល្អ និង ទទួលបានជ័យជំនះក្នុងការសិក្សា ។

បាត់ដំបងថ្ងៃទី ២៣ ខែ តុលា ឆ្នាំ២០០៩

អ្នកនិពន្ធ **លឹម ផល្លុន**

ជំពូកទី១

លីមីតនៃអនុគមន៍

១/សញ្ញាណលីមីត

បើ x ខិតជិត a ហើយអនុគមន៍ខិតជិតតម្លៃ L ណាមួយ នោះគេកំនត់សរសេរ $\lim_{x\to a} f(x)=L$

មានន័យថាអនុគមន៍ f(x) មានលីមីតស្មើ L កាលណា x ខិតជិត a ។

២/ប្រមាណវិធីលើលីមីត

ឧបមាថាអនុគមន៍ $y=f(x)$ និង $y=g(x)$ មានលីមីត កាលណា $x\to a$ នោះគេមានប្រមាណវិធីលីមីតដូចខាងក្រោម

ក/ $\lim_{x\to a} f(x)=\lim_{x\to a} k=k$ ដែល $f(x)=k$ អនុគមន៍ថេរ

ខ/ $\lim_{x\to a}\left[k f(x)\right]=k\lim_{x\to a} f(x)$

គ/ $\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$

ឃ/ $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a} f(x)-\lim_{x\to a} g(x)$

ង/ $\lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a} f(x)\times\lim_{x\to a} g(x)$

ឋ/ $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)}$ ដែល $\lim_{x\to a}g(x)\neq 0$

៣/លីមីតខាងឆ្វេងនិងខាងស្តាំ

-បើអនុគមន៍ $f(x)$ ខិតជិត L កាលណា x ខិតជិត x_0 ពីខាងឆ្វេងនោះ L ជាលីមីតខាងឆ្វេងនៃ $f(x)$ ហើយគេកំនត់សរសេរ $\lim_{x\to x_0^-}f(x)=R$

-បើអនុគមន៍ $f(x)$ ខិតជិត R កាលណា x ខិតជិត x_0 ពីខាងស្តាំនោះ R ជាលីមីតខាងស្តាំនៃ $f(x)$ ហើយគេកំនត់សរសេរ $\lim_{x\to x_0^+}f(x)=R$

-អនុគមន៍ $y=f(x)$ មានលីមីតត្រង់ x_0 លុះត្រាតែលីមីតខាងឆ្វេង ស្មើនឹងលីមីតខាងស្តាំ ។

៤/លីមីតអនន្តនៃអនុគមន៍

ក/ $\lim_{x\to\pm\infty}ax^2=\begin{cases}+\infty & \text{បើ } a>0\\ -\infty & \text{បើ } a<0\end{cases}$

ខ/ $\lim_{x\to\pm\infty}\frac{a}{x}=0$, $a\in IR$

៥/លីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

ក/ $\lim\limits_{x\to0}\frac{\sin x}{x}=\lim\limits_{x\to0}\frac{x}{\sin x}=1$

ខ/ $\lim\limits_{x\to0}\frac{\tan x}{x}=\lim\limits_{x\to0}\frac{x}{\tan x}=1$

គ/ $\lim\limits_{x\to0}\frac{1-\cos x}{x}=0$

៦/លីមីតនៃអនុគមន៍អិចស្ប៉ូណង់ស្យែល

ក/ $\lim\limits_{x\to0}\frac{e^x-1}{x}=1$

ខ/ $\lim\limits_{x\to0}\frac{a^x-1}{x}=\ln a \quad ,(a>0)$

គ/ $\lim\limits_{x\to0}\left(1+x\right)^{\frac{1}{x}}=\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

ឃ/ $\lim\limits_{x\to+\infty}a^x=\begin{cases}+\infty & \text{បើ } a>1\\ 0 & \text{បើ } 0<a<1\end{cases}$

៧/លីមីតនៃអនុគមន៍លោការីត

ក/ $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1$

ខ/ $\lim\limits_{x\to0^+}\ln x=-\infty$

គ/ $\lim\limits_{x\to+\infty}\ln x=+\infty$

ជំពូកទី២

វិធីសាស្ត្រគណនាលីមីត

១/របៀបគណនាលីមីតតាមការដាក់ជាផលគុណកត្តា

ឧបមាថាគេមានលីមីត $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ (1)

បើតម្លៃលេខ $g(a)=0$ និង $f(a)=0$ នោះលីមីត(1) មានរាងមិនកំនត់ $\frac{0}{0}$ ។ ក្នុងករណីនេះដើម្បីគណនាលីមីតគេត្រូវដាក់ភាគយកនិងភាគបែងជាផលគុណនៃកត្តាដោយមាន $(x-a)$ ជាកត្តារួម រួចសម្រួលកត្តារួមនេះចោលបន្ទាប់មករកលីមីតនៃប្រភាគថ្មី ។

ឧទាហរណ៍គំរូ ចូរគណនាលីមីតខាងក្រោម

១/ $\lim\limits_{x\to 2}\frac{5x^2-8x-4}{x^3-8}$

យើងសង្កេតឃើញថាលីមីតនេះមានរាងមិនកំនត់ $\frac{0}{0}$

យើងមាន $5x^2-8x-4=(5x^2-10x)+(2x-4)$

$$=(x-2)(5x-2)$$

និង $x^3-8=(x-2)(x^2+2x+4)$

យើងបាន

$$\lim_{x\to 2}\frac{5x^2-8x-4}{x^3-8}=\lim_{x\to 2}\frac{(x-2)(5x+2)}{(x-2)(x^2+2x+4)}$$

$$=\lim_{x\to 2}\frac{5x+2}{x^2+2x+4}$$

$$=\frac{5(2)+2}{(2)^2+2(2)+4}=\frac{12}{12}=1$$

ដូចនេះ: $\boxed{\lim_{x\to 2}\frac{5x^2-8x-4}{x^3-8}=1}$

២/ $\lim_{x\to 3}\frac{x^3-3x^2+x-3}{2x^2-7x+3}$ (មានរាងមិនកំនត់ $\frac{0}{0}$)

$$=\lim_{x\to 3}\frac{(x^3-3x^2)+(x-3)}{(2x^2-6x)-(x-3)}$$

$$=\lim_{x\to 3}\frac{x^2(x-3)+(x-3)}{2x(x-3)-(x-3)}$$

$$=\lim_{x\to 3}\frac{(x-3)(x^2+1)}{(x-3)(2x-1)}$$

$$=\lim_{x\to 3}\frac{x^2+1}{2x-1}=\frac{3^2+1}{2(3)-1}=2$$

ដូចនេះ: $\boxed{\lim_{x\to 3}\frac{x^3-3x^2+x-3}{2x^2-7x+3}=2}$

៣/ $\lim_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}$

$=\lim_{x\to 1}\frac{(3x^4-3x^3)-(x^3-1)}{(x-1)^2}$

$=\lim_{x\to 1}\frac{3x^3(x-1)-(x-1)(x^2+x+1)}{(x-1)^2}$

$=\lim_{x\to 1}\frac{3x^3-x^2-x-1}{x-1}$

$=\lim_{x\to 1}\frac{(x^3-x^2)+(x^3-x)+(x^3-1)}{x-1}$

$=\lim_{x\to 1}\frac{x^2(x-1)+x(x-1)(x+1)+(x-1)(x^2+x+1)}{x-1}$

$=\lim_{x\to 1}\left[x^2+x(x+1)+(x^2+x+1)\right]=1+2+3=6$

ដូចនេះ: $\boxed{\lim_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}=6}$ ។

៤/ $\lim_{x\to 1}\frac{x^4-4x+3}{x^3-x^2-x+1}$

$=\lim_{x\to 1}\frac{(x^4-x)-3(x-1)}{x^2(x-1)-(x-1)}$

$=\lim_{x\to 1}\frac{x(x-1)(x^2+x+1)-3(x-1)}{(x-1)(x^2-1)}$

$$=\lim_{x\to1}\frac{(x^4-x)-3(x-1)}{x^2(x-1)-(x-1)}$$

$$=\lim_{x\to1}\frac{x(x-1)(x^2+x+1)-3(x-1)}{(x-1)(x^2-1)}$$

$$=\lim_{x\to1}\frac{x^3+x^2+x-3}{x^2-1}$$

$$=\lim_{x\to1}\frac{(x^3-1)+(x^2-1)+(x-1)}{(x-1)(x+1)}$$

$$=\lim_{x\to1}\frac{(x^2+x+1)+(x+1)+1}{x+1}=\frac{3+2+1}{2}=\frac{6}{2}=3$$

ដូចនេះ: $\boxed{\lim_{x\to1}\frac{x^4-4x+3}{x^3-x^2-x+1}=3}$ ។

ជ/ $\lim_{x\to0}\frac{(1+ax)^n-1}{x}$

$$=\lim_{x\to0}\frac{\left[(1+ax)-1\right]\left[(1+ax)^{n-1}+....+(1+ax)+1\right]}{x}$$

$$=\lim_{x\to0}a\left[(1+ax)^{n-1}+....+(1+ax)+1\right]$$

$$=a\left(1+1+.....+1\right)=n.a$$

ដូចនេះ: $\boxed{\lim_{x\to0}\frac{(1+ax)^n-1}{x}=na}$ ។

៦/$\lim_{x\to 0}\frac{(1+x)^3-(3x+1)}{x^2}$

$=\lim_{x\to 0}\frac{1+3x+3x^2+x^3-3x-1}{x^2}$

$=\lim_{x\to 0}\frac{3x^2+x^3}{x^2}=\lim_{x\to 0}(3+x)=3$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{(1+x)^3-(3x+1)}{x^2}=3}$ ។

៧/. $\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)}{x}$

$=\lim_{x\to 0}\frac{1-(1+x)+(1+x)[1-(1+2x)]+(1+x)(1+2x)[1-(1+3x)]}{x}$

$=\lim_{x\to 0}\frac{-x-2x(1+x)-3x(1+x)(1+2x)}{x}$

$=-\lim_{x\to 0}[1+2(1+x)+3(1+x)(1+2x)]=-(1+2+3)=-6$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)}{x}=-6}$ ។

៨/$\lim_{x\to 1}\frac{x^n-1}{x-1}$ ដែល $n\in \mathbb{N}^*$

$=\lim_{x\to 1}\frac{(x-1)(x^{n-1}+....+x+1)}{x-1}$

$=\lim_{x\to 1}(x^{n-1}+....+x+1)=1+1+...+1=n$

$$៩/\lim_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$$

$$=\lim_{x\to1}\left[\frac{1}{1-x}-\frac{3}{(1-x)(1+x+x^2)}\right]$$

$$=\lim_{x\to1}\left[\frac{(1+x+x^2)-3}{(1-x)(1+x+x^2)}\right]$$

$$=\lim_{x\to1}\frac{(x-1)+(x^2-1)}{(1-x)(1+x+x^2)}$$

$$=\lim_{x\to1}\frac{(x-1)+(x-1)(x+1)}{-(x-1)(x^2+x+1)}$$

$$=\lim_{x\to1}\frac{1+(x+1)}{-(x^2+x+1)}=\frac{1+2}{-3}=-1$$

ដូចនេះ: $\boxed{\lim_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=-1}$ ។

$$១០/\lim_{x\to2}\frac{x^6-x^2-60}{x^2-4}$$

$$=\lim_{x\to2}\frac{(x^2-4)(x^4+4x^2+15)}{x^2-4}$$

$$=\lim_{x\to2}(x^4+4x^2+15)=16+16+15=47$$

ដូចនេះ: $\boxed{\lim_{x\to2}\frac{x^6-x^2-60}{x^2-4}=47}$

១១/ $\lim_{x\to1}\frac{x+x^2+x^3+....+x^n-n}{x+x^2+x^3+....+x^p-p}$, $(n\in IN^*, p\in IN^*)$

$$=\lim_{x\to1}\frac{(x-1)+(x^2-1)+...+(x^n-1)}{(x-1)+(x^2-1)+...+(x^p-1)}$$

$$=\lim_{x\to1}\frac{(x-1)+(x-1)(x+1)+...+(x-1)(x^{n-1}+...+x+1)}{(x-1)+(x-1)(x+1)+...+(x-1)(x^{p-1}+...+x+1)}$$

$$=\lim_{x\to1}\frac{(x-1)\left[1+(x+1)+....+(x^{n-1}+....+x+1)\right]}{(x-1)\left[1+(x+1)+....+(x^{p-1}+....+x+1)\right]}$$

$$=\lim_{x\to1}\frac{1+(x+1)+....+(x^{n-1}+....+x+1)}{1+(x+1)+....+(x^{p-1}+....+x+1)}$$

$$=\frac{1+2+3+......+n}{1+2+3+......+p}=\frac{\frac{n(n+1)}{2}}{\frac{p(p+1)}{2}}=\frac{n(n+1)}{p(p+1)}$$

ដូចនេះ: $\boxed{\lim_{x\to1}\frac{x+x^2+x^3+....+x^n-n}{x+x^2+x^3+....+x^p-p}=\frac{n(n+1)}{p(p+1)}}$

១២/ $\lim_{x\to1}\frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}$, $n\in IN^*$

$$=\lim_{x\to1}\frac{nx^n(x-1)-(x-1)(x^{n-1}+....+x^2+x+1)}{(x-1)^2}$$

$$=\lim_{x\to1}\frac{(x^n-x^{n-1})+....+(x^n-x)+(x^n-1)}{x-1}$$

$$=\lim_{x\to1}\frac{x^{n-1}(x-1)+...+(x-1)(x^{n-1}+...+x+1)}{x-1}$$

$$=\lim_{x\to1}\frac{x^{n-1}(x-1)+...+(x-1)(x^{n-1}+...+x+1)}{x-1}$$

$$=\lim_{x\to1}\left[x^{n-1}+....+...(x^{n-1}+...+x+1)\right]$$

$$=1+2+....+(n-1)+n=\frac{n(n+1)}{2}$$

ដូចនេះ: $\boxed{\lim_{x\to1}\frac{nx^{n+1}-(n+1)x^{n}+1}{(x-1)^{2}}=\frac{n(n+1)}{2}}$

១៣/. $\lim_{x\to1}\frac{x^{n+1}-(n+1)x+n}{x^{p+1}-x^{p}-x+1}, n,p\in IN^*$

$$=\lim_{x\to1}\frac{(x^{n+1}-x)-n(x-1)}{x^{p}(x-1)-(x-1)}$$

$$=\lim_{x\to1}\frac{x(x-1)(x^{n-1}+...+x+1)-n(x-1)}{(x-1)(x^{p}-1)}$$

$$=\lim_{x\to1}\frac{x^{n}+....+x^{2}+x-n}{x^{p}-1}$$

$$=\lim_{x\to1}\frac{(x^{n}-1)+....+(x^{2}-1)+(x-1)}{x^{p}-1}$$

$$=\lim_{x\to1}\frac{(x-1)(x^{n-1}+...+x+1)+...+(x-1)(x+1)+(x-1)}{(x-1)(x^{p-1}+....+x+1)}$$

$$=\lim_{x\to1}\frac{(x^{n-1}+.....+x+1)+....+(x+1)+1}{x^{p-1}+....+x+1}$$

$$=\frac{n+....+2+1}{p}=\frac{n(n+1)}{2p}$$

២/របៀបគណនាលីមីតអនុគមន៍អសនិទាន

ក/ករណីកន្សោមរាង $\sqrt{A}-B$

គេមាន $(\sqrt{A}-B)(\sqrt{A}+B)=A-B^2$

គេបាន $\boxed{\sqrt{A}-B=\dfrac{A-B^2}{\sqrt{A}+B}}$

<u>ឧទាហរណ៍</u> គណនាលីមីត

១/$\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-3}{x^2-2x}$

គេមាន $\sqrt{x^3+1}-3=\dfrac{(x^3+1)-3^2}{\sqrt{x^3+1}+3}$

$$=\frac{x^3-8}{\sqrt{x^3+1}+3}$$

$$=\frac{(x-2)(x^2+2x+4)}{\sqrt{x^3+1}+3}$$

$$\lim_{x\to 2}\frac{\sqrt{x^3+1}-3}{x^2-2x}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x(x-2)(\sqrt{x^3+1}+3)}$$

$$=\lim_{x\to 2}\frac{x^2+2x+4}{x(\sqrt{x^3+1}+3)}=\frac{12}{12}=1$$

ដូចនេះ: $\boxed{\lim\limits_{x\to 2}\dfrac{\sqrt{x^3+1}-3}{x^2-2x}=1}$

$$២/\lim_{x\to 3}\frac{x^3-27}{\sqrt{x^2-5}-2}$$

$$=\lim_{x\to 3}\frac{(x^3-27)(\sqrt{x^2-5}+2)}{x^2-5-4}$$

$$=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)(\sqrt{x^2-5}+2)}{(x-3)(x+3)}$$

$$=\lim_{x\to 3}\frac{(x^2+3x+9)(\sqrt{x^2-5}+2)}{x+3}=\frac{27\times 4}{6}=18$$

ដូចនេះ: $\boxed{\lim_{x\to 3}\frac{x^3-27}{\sqrt{x^2-5}-2}=18}$

$$៣/\lim_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$$

$$=\lim_{x\to 2}\frac{(x^2-3-1)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)(x+2-4)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+2)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)(x-2)}$$

$$=\lim_{x\to 2}\frac{(x+2)(\sqrt{x+2}+2)}{\sqrt{x^2-3x}+1}=\frac{4\times 4}{2}=8$$

ដូចនេះ: $\boxed{\lim_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}=8}$

ខ/ករណីកន្សោមរាង $A-\sqrt{B}$

គេមាន $(A-\sqrt{B})(A+\sqrt{B})=A^2-B$

គេបាន $\boxed{A-\sqrt{B}=\dfrac{A^2-B}{A+\sqrt{B}}}$

<u>ឧទាហរណ៍</u> គណនាលីមីត

១/$\lim\limits_{x\to 3}\dfrac{x-\sqrt{2x+3}}{x-3}$

$$=\lim_{x\to 3}\frac{x^2-(2x+3)}{(x-3)(x+\sqrt{2x+3})}$$

$$=\lim_{x\to 3}\frac{x^2-2x-3}{(x-3)(x+\sqrt{2x+3})}$$

$$=\lim_{x\to 3}\frac{(x+1)(x-3)}{(x-3)(x+\sqrt{2x+3})}$$

$$=\lim_{x\to 3}\frac{x+1}{x+\sqrt{2x+3}}=\frac{4}{6}=\frac{2}{3}$$

ដូចនេះ: $\lim\limits_{x\to 3}\dfrac{x-\sqrt{2x+3}}{x-3}$

២/$\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x-\sqrt{2x-1}}$

$$=\lim_{x\to 1}\frac{(x-1)^2(x+\sqrt{2x-1})}{x^2-(2x-1)}$$

$$=\lim_{x\to 1}\frac{(x-1)^2(x+\sqrt{2x-1})}{x^2-2x+1}$$

$$=\lim_{x\to 1}\frac{(x-1)^2(x+\sqrt{2x-1})}{(x-1)^2}$$

$$=\lim_{x\to 1}(x+\sqrt{2x-1})=1+1=2$$

ដូចនេះ: $\lim_{x\to 1}\frac{(x-1)^2}{x-\sqrt{2x-1}}=2$

៣/$\lim_{x\to 4}\frac{\sqrt{x^2+9}-5}{\sqrt{x}-2}$

$$=\lim_{x\to 4}\frac{(x^2+9-25)(\sqrt{x}+2)}{(x-4)(\sqrt{x^2+9}+5)}$$

$$=\lim_{x\to 4}\frac{(x-4)(x+4)(\sqrt{x}+2)}{(x-4)(\sqrt{x^2+9}+5)}$$

$$=\lim_{x\to 4}\frac{(x+4)(\sqrt{x}+2)}{\sqrt{x^2+9}+5}=\frac{8\times 4}{10}=\frac{16}{5}$$

ដូចនេះ: $\lim_{x\to 4}\frac{\sqrt{x^2+9}-5}{\sqrt{x}-2}$

គ/ករណីកន្សោមរាង $\sqrt{A}-\sqrt{B}$

គេមាន $(\sqrt{A}-\sqrt{B})(\sqrt{A}+\sqrt{B})=A-B$

គេបាន $\boxed{\sqrt{A}-\sqrt{B}=\frac{A-B}{\sqrt{A}+\sqrt{B}}}$

ឧទាហរណ៍ គណនាលីមីត

១/ $\lim_{x\to 0}\frac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}$

$$=\lim_{x\to 0}\frac{(x^2-x+1)-(x^2+x+1)}{x(\sqrt{x^2-x+1}+\sqrt{x^2+x+1})}$$

$$=\lim_{x\to 0}\frac{-2x}{x(\sqrt{x^2-x+1}+\sqrt{x^2+x+1})}$$

$$=\lim_{x\to 0}\frac{-2}{\sqrt{x^2-x+1}+\sqrt{x^2+x+1}}=\frac{-2}{2}=-1$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{\sqrt{x^2-x+1}-\sqrt{x^2+x+1}}{x}=-1}$

២/ $\lim_{x\to 3}\frac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}$

$$=\lim_{x\to 3}\frac{(x^2-5x+6)(\sqrt{2x+3}+\sqrt{x+6})}{(2x+3)-(x+6)}$$

$$=\lim_{x\to 3}\frac{(x-2)(x-3)(\sqrt{2x+3}+\sqrt{x+6})}{x-3}$$

$$=\lim_{x\to 3}\left[(x-2)(\sqrt{2x+3}+\sqrt{x+6})\right]=6$$

ដូចនេះ: $\boxed{\lim_{x\to 3}\frac{x^2-5x+6}{\sqrt{2x+3}-\sqrt{x+6}}=6}$

៣/ $\lim_{x\to 2}\frac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}$

$$=\lim_{x\to 2}\frac{(x^2-2-4x+6)(\sqrt{x}+\sqrt{2})^2}{(x-2)^2(\sqrt{x^2-2}+\sqrt{4x-6})}$$

$$=\lim_{x\to 2}\frac{(x-2)^2(\sqrt{x}+\sqrt{2})^2}{(x-2)^2(\sqrt{x^2-2}+\sqrt{4x-6})}$$

$$=\lim_{x\to 2}\frac{(\sqrt{x}+\sqrt{2})^2}{\sqrt{x^2-2}+\sqrt{4x-6}}=\frac{(2\sqrt{2})^2}{2\sqrt{2}}=2\sqrt{2}$$

ដូចនេះ: $\boxed{\lim_{x\to 2}\frac{\sqrt{x^2-2}-\sqrt{4x-6}}{(\sqrt{x}-\sqrt{2})^2}=2\sqrt{2}}$

៤/. $\lim_{x\to 1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$

$$=\lim_{x\to 1}\frac{\left(x^2+3x-3x-1\right)}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x^2+3x}+\sqrt{3x+1}}$$

$$=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x^2+3x}+\sqrt{3x+1}}$$

$$=\lim_{x\to 1}\frac{(x+1)(\sqrt{x}+1)}{\sqrt{x^2+3x}+\sqrt{3x+1}}=\frac{2.2}{2+2}=1$$

ដូចនេះ: $\boxed{\lim_{x\to 1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}=1}$

ឃ/ករណីកន្សោមរាង $\sqrt[3]{A}-B$

គេមាន $(\sqrt[3]{A}-B)(\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2)=A-B^3$

គេបាន $$\boxed{\sqrt[3]{A}-B=\frac{A-B^3}{\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2}}$$

<u>**ឧទាហរណ៍**</u> គណនាលីមីត

១/ $\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$

$=\lim\limits_{x\to 2}\frac{x-8}{(x-8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}$

$=\lim\limits_{x\to 2}\frac{1}{\sqrt[3]{x^2}+2\sqrt[3]{x}+4}=\frac{1}{4+4+4}=\frac{1}{12}$

ដូចនេះ: $\boxed{\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\frac{1}{12}}$

២/$\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$

$=\lim\limits_{x\to 0}\frac{(1+x^2)-1}{x^2\left(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1\right)}$

$=\lim\limits_{x\to 0}\frac{1}{\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1}=\frac{1}{3}$

ដូចនេះ: $\boxed{\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}=\frac{1}{3}}$

ង/ករណីកន្សោមរាង $A-\sqrt[3]{B}$

គេមាន $(A-\sqrt[3]{B})(A^2+A\sqrt[3]{B}+\sqrt[3]{B^2})=A^3-B$

គេបាន $$\boxed{A-\sqrt[3]{B}=\frac{A^3-B}{A^2+A\sqrt[3]{B}+\sqrt[3]{B^2}}}$$

<u>ឧទាហរណ៍</u> គណនាលីមីត

១/ $\lim\limits_{x\to2}\dfrac{x-\sqrt[3]{x^2+4}}{x-2}$

$$=\lim_{x\to2}\frac{x^3-(x^2+4)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to2}\frac{x^3-x^2-4}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to2}\frac{(x^3-2x^2)+(x^2-4)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to2}\frac{(x-2)(x^2+x+2)}{(x-2)(x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2})}$$

$$=\lim_{x\to2}\frac{x^2+x+2}{x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}}=\frac{8}{12}=\frac{2}{3}$$

ដូចនេះ: $$\boxed{\lim_{x\to2}\frac{x-\sqrt[3]{x^2+4}}{x-2}=\frac{2}{3}}$$

២/$\lim\limits_{x\to 2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

$=\lim\limits_{x\to 2}\frac{x^6-x^2-60}{x^6-x^2-60}\cdot\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}}{x^3+\sqrt{x^2+60}}$

$=\lim\limits_{x\to 2}\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}}{x^3+\sqrt{x^2+60}}$

$=\frac{16+16+16}{8+8}=\frac{48}{16}=3$

ដូចនេះ: $\boxed{\lim\limits_{x\to 2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}=3}$ ។

៣/$\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}$

$=\lim\limits_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}]}{(x-1)^3-(x^2-1)}$

$=\lim\limits_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}]}{x(x-1)(x-3)}$

$=\lim\limits_{x\to 3}\frac{(x-1)^2+(x-1)\sqrt[3]{x^2-1}+\sqrt[3]{(x^2-1)^2}}{x(x-1)}=\frac{12}{6}=2$

ដូចនេះ: $\boxed{\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}=2}$

ច/ករណីកន្សោមរាង $\sqrt[3]{A}-\sqrt[3]{B}$

គេមាន $(\sqrt[3]{A}-\sqrt[3]{B})(\sqrt[3]{A^2}+\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2})=A-B$

គេបាន $$\boxed{\sqrt[3]{A}-\sqrt[3]{B}=\frac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}}$$

ឧទាហរណ៍ ចូរគណនាលីមីត

១/ $\lim\limits_{x\to 3}\frac{\sqrt[3]{x^2-1}-\sqrt[3]{x+5}}{x-3}$

$$=\lim_{x\to 3}\frac{(x^2-1)-(x+5)}{(x-3)[\sqrt[3]{(x^2-1)^2}+\sqrt[3]{(x^2-1)(x+5)}+\sqrt[3]{(x+5)^2}]}$$

$$=\lim_{x\to 3}\frac{x^2-x-6}{(x-3)[\sqrt[3]{(x^2-1)^2}+\sqrt[3]{(x^2-1)(x+5)}+\sqrt[3]{(x+5)^2}]}$$

$$=\lim_{x\to 3}\frac{(x+2)(x-3)}{(x-3)[\sqrt[3]{(x^2-1)^2}+\sqrt[3]{(x^2-1)(x+5)}+\sqrt[3]{(x+5)^2}]}$$

$$=\lim_{x\to 3}\frac{x+2}{\sqrt[3]{(x^2-1)^2}+\sqrt[3]{(x^2-1)(x+5)}+\sqrt[3]{(x+5)^2}}$$

$$=\frac{5}{4+4+4}=\frac{5}{12}$$

ដូចនេះ $\lim\limits_{x\to 3}\frac{\sqrt[3]{x^2-1}-\sqrt[3]{x+5}}{x-3}=\frac{5}{12}$

២/ $\lim_{x\to 1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}$

$$=\lim_{x\to 1}\frac{(x^3+7x)-(10x-2)}{(x-1)^2[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}]}$$

$$=\lim_{x\to 1}\frac{x^3-3x+2}{(x-1)^2[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}]}$$

$$=\lim_{x\to 1}\frac{(x-1)^2(x+2)}{(x-1)^2[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}]}$$

$$=\lim_{x\to 1}\frac{x+2}{\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}}$$

$$=\frac{3}{4+4+4}=\frac{1}{4}$$

ដូចនេះ: $\lim_{x\to 1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}=\frac{1}{4}$

៣/ $\lim_{x\to 2}\frac{\sqrt[3]{x^2+4}-\sqrt[3]{x+6}}{x-2}$

$$=\lim_{x\to 2}\frac{(x^2+4)-(x+6)}{(x-2)[\sqrt[3]{(x^2+4)^2}+\sqrt[3]{(x^2+4)(x+6)}+\sqrt[3]{(x+6)^2}]}$$

$$=\lim_{x\to 2}\frac{(x+1)(x-2)}{(x-2)[\sqrt[3]{(x^2+4)^2}+\sqrt[3]{(x^2+4)(x+6)}+\sqrt[3]{(x+6)^2}]}$$

$$=\lim_{x\to 2}\frac{x+1}{\sqrt[3]{(x^2+4)^2}+\sqrt[3]{(x^2+4)(x+6)}+\sqrt[3]{(x+6)^2}}=\frac{3}{12}=\frac{1}{4}$$

ដូចនេះ: $\lim_{x\to 2}\frac{\sqrt[3]{x^2+4}-\sqrt[3]{x+6}}{x-2}=\frac{1}{4}$

ឆ/ករណីកន្សោមរាង $\sqrt[3]{A}+\sqrt[3]{B}$

គេមាន $(\sqrt[3]{A}+\sqrt[3]{B})(\sqrt[3]{A^2}-\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2})=A+B$

គេបាន $\boxed{\sqrt[3]{A}+\sqrt[3]{B}=\dfrac{A+B}{\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}}$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

$$=\lim_{x\to 0}\frac{(x^2+1)+(x^2-1)}{x^2[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}$$

$$=\lim_{x\to 0}\frac{2x^2}{x^2[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}$$

$$=\lim_{x\to 0}\frac{2}{\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}}$$

$$=\frac{2}{1+1+1}=\frac{2}{3}$$

ដូចនេះ: $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}=\dfrac{2}{3}$

២/ $\lim\limits_{x\to 4}\dfrac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$

$$=\lim_{x\to 4}\frac{(x+4)+(8-x^2)}{(x-4)[\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}]}$$

$$=\lim_{x\to 4}\frac{12+x-x^2}{(x-4)[\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}]}$$

$$=\lim_{x\to 4}\frac{(4-x)(3+x)}{(x-4)[\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}]}$$

$$=\lim_{x\to 4}\frac{-(x+3)}{\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}}$$

$$=\frac{-7}{4+4+4}=-\frac{7}{12}$$

ដូចនេះ: $\lim_{x\to 4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$ ។

៣/ $\lim_{x\to -2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$

$$=\lim_{x\to -2}\frac{(x^2-2x)+(x-6)}{(x+2)[\sqrt[3]{x^2-2x}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}]}$$

$$=\lim_{x\to -2}\frac{(x+2)(x-3)}{(x+2)[\sqrt[3]{x^2-2x}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}]}$$

$$=\lim_{x\to -2}\frac{x-3}{\sqrt[3]{x^2-2x}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}}$$

$$=\frac{-5}{4+4+4}=-\frac{5}{12}$$

ដូចនេះ: $\lim_{x\to -2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}=-\frac{5}{12}$

៣/របៀបគណនាលីមីតត្រង់អនន្ត

៦. រូបមន្តសំខាន់ៗ

ក/ $\lim_{x\to\pm\infty} ax^2 = \begin{cases} +\infty & \text{បើ } a>0 \\ -\infty & \text{បើ } a<0 \end{cases}$

ខ/ $\lim_{x\to\pm\infty} \frac{a}{x} = 0 \quad , \; a\in IR$

៦. របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមរាង

$$L = \lim_{x\to\infty} \frac{a_m x^m + a_{m-1}x^{m-1} + \dots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \dots + b_1 x + b_0}$$

ដើម្បីគណនាលីមីតនេះគេត្រូវ

-ដាក់តួ x ដែលមានស្វ័យគុណខ្ពស់ជាងគេជាកត្តារួម

-សម្រួលកត្តា x នោះចោល

-គណនាលីមីតដោយប្រើរូបមន្ត $\lim_{x\to\infty}\frac{1}{x^\alpha} = 0, \alpha > 0$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim_{x\to\infty}\frac{4x^3 - 7x^2 + x + 3}{6x^3 + x^2 - 5x + 1}$

$$= \lim_{x\to\infty}\frac{x^3(4 - \frac{7}{x} + \frac{1}{x^2} + \frac{3}{x^3})}{x^3(6 + \frac{1}{x} - \frac{5}{x^2} + \frac{1}{x^3})}$$

$$=\lim_{x\to\infty}\frac{4-\frac{7}{x}+\frac{1}{x^2}+\frac{3}{x^3})}{6+\frac{1}{x}-\frac{5}{x^2}+\frac{1}{x^3}}=\frac{4}{6}=\frac{2}{3}$$

ដូចនេះ: $\lim_{x\to\infty}\frac{4x^3-7x^2+x+3}{6x^3+x^2-5x+1}$

២/ $\lim_{x\to+\infty}\frac{x^{100}+(x+1)^{100}+....+(x+100)^{100}}{x^{100}+100^{100}}$

$$=\lim_{x\to+\infty}\frac{x^{100}+x^{100}(1+\frac{1}{x})^{100}+....+x^{100}(1+\frac{100}{x})^{100}}{x^{100}+100^{100}}$$

$$=\lim_{x\to\infty}\frac{x^{100}\left[1+(1+\frac{1}{x})^{100}+....+(1+\frac{100}{x})^{100}\right]}{x^{100}\left(1+\frac{100^{100}}{x^{100}}\right)}$$

$$=\lim_{x\to\infty}\frac{1+(1+\frac{1}{x})^{100}+....+(1+\frac{100}{x})^{100}}{1+\frac{100^{100}}{x^{100}}}$$

$$=\frac{1+1+....+1}{1}=101$$

ដូចនេះ: $\lim_{x\to+\infty}\frac{x^{100}+(x+1)^{100}+....+(x+100)^{100}}{x^{100}+100^{100}}=101$

៦. របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមអសនិទាន

គេត្រូវប្រើរូបមន្តបំលែងដូចខាងក្រោម

1/ $\sqrt{A}-\sqrt{B}=\dfrac{A-B}{\sqrt{A}+\sqrt{B}}$

2/ $\sqrt[3]{A}-\sqrt[3]{B}=\dfrac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2}}$

3/ $\sqrt[3]{A}+\sqrt[3]{B}=\dfrac{A+B}{\sqrt[3]{A^2}-\sqrt[3]{A}.\sqrt[3]{B}+\sqrt[3]{B^2}}$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim\limits_{x\to+\infty}(\sqrt{x^2+5x}-\sqrt{x^2-3x+1})$

$$=\lim_{x\to+\infty}\frac{(x^2+5x)-(x^2-3x+1)}{\sqrt{x^2+5x}+\sqrt{x^2-3x+1}}$$

$$=\lim_{x\to+\infty}\frac{8x-1}{|x|\sqrt{1+\frac{5}{x}}+|x|\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}}$$

$$=\lim_{x\to+\infty}\frac{x(8-\frac{1}{x})}{x(\sqrt{1+\frac{5}{x}}+\sqrt{1-\frac{3}{x}+\frac{1}{x^2}})}$$

$$=\lim_{x\to+\infty}\frac{8-\frac{1}{x}}{\sqrt{1+\frac{5}{x}}+\sqrt{1-\frac{3}{x}+\frac{1}{x^2}}}=\frac{8}{2}=4$$

ដូចនេះ $\lim_{x\to+\infty}(\sqrt{x^2+5x}-\sqrt{x^2-3x+1})=4$

២/ $\lim_{x\to-\infty}(\sqrt{4x^2+7x}-\sqrt{4x^2-5x+3})$

$$=\lim_{x\to-\infty}\frac{(4x^2+7x)-(4x^2-5x+3)}{\sqrt{4x^2+7x}+\sqrt{4x^2-5x+3}}$$

$$=\lim_{x\to-\infty}\frac{12x-3}{|x|\sqrt{4+\frac{7}{x}}+|x|\sqrt{4-\frac{5}{x}+\frac{3}{x^2}}}$$

$$=\lim_{x\to-\infty}\frac{x(12-\frac{3}{x})}{-x(\sqrt{4+\frac{7}{x}}+\sqrt{4-\frac{5}{x}+\frac{3}{x^2}})}$$

$$=\lim_{x\to-\infty}\frac{12-\frac{3}{x}}{-(\sqrt{4+\frac{7}{x}}+\sqrt{4-\frac{5}{x}+\frac{3}{x^2}}}=-\frac{12}{4}=-3$$

ដូចនេះ $\lim_{x\to-\infty}(\sqrt{4x^2+7x}-\sqrt{4x^2-5x+3})=-3$

កំនត់សម្គាល់

-បើ $x\to+\infty$ នោះគេត្រូវបញ្ចេញ $|x|=x$

-បើ $x\to-\infty$ នោះគេត្រូវបញ្ចេញ $|x|=-x$

៣/ $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

$$=\lim_{x\to+\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x(1+\frac{\sqrt{x}}{x})}}{\sqrt{x(1+\frac{\sqrt{x+\sqrt{x}}}{x})}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+\sqrt{x}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1}=\frac{1}{1+1}=\frac{1}{2}$$

ដូចនេះ: $\boxed{\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=\frac{1}{2}}$

ជ/ $\lim_{x\to+\infty}\left(\sqrt{x^2+7x+1}-x\right)$

$$=\lim_{x\to+\infty}\frac{x^2+7x+1-x^2}{\sqrt{x^2+7x+1}+x}$$

$$=\lim_{x\to+\infty}\frac{7x+1}{\sqrt{x^2(1+\frac{7}{x}+\frac{1}{x^2})}+x}$$

$$=\lim_{x\to+\infty}\frac{x(7+\frac{1}{x})}{x(\sqrt{1+\frac{7}{x}+\frac{1}{x^2}}+1)}=\lim_{x\to+\infty}\frac{7+\frac{1}{x}}{\sqrt{1+\frac{7}{x}+\frac{1}{x^2}}+1}=\frac{7}{2}$$

ដូចនេះ: $\boxed{\lim_{x\to+\infty}\left(\sqrt{x^2+7x+1}-x\right)=\frac{7}{2}}$

ឈ/ $\lim_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2+1}-x\right)$

$$=\lim_{x\to\infty}\frac{x^3+3x^2+1-x^3}{\sqrt[3]{(x^3+3x^2+1)^2}+x\sqrt[3]{x^3+3x^2+1}+x^2}$$

$$=\lim_{x\to\infty}\frac{3x^2+1}{\sqrt[3]{x^6(1+\frac{3}{x}+\frac{1}{x^3})^2}+x\sqrt[3]{x^3(1+\frac{3}{x}+\frac{1}{x^3})}+x^2}$$

$$=\lim_{x\to\infty}\frac{x^2(3+\frac{1}{x^2})}{x^2[\sqrt[3]{(1+\frac{3}{x}+\frac{1}{x^3})^2}+\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}}+1]}$$

$$=\lim_{x\to\infty}\frac{3+\frac{1}{x^2}}{\sqrt[3]{(1+\frac{3}{x}+\frac{1}{x^3})^2}+\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}}+1}=\frac{3}{1+1+1}=1$$

ដូចនេះ: $\lim_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2+1}-x\right)=1$

៦/ $\lim_{x\to\infty}[\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}]$

$$=\lim_{x\to\infty}\frac{x^3+6x^2-x^3+6x^2}{\sqrt[3]{(x^3+6x^2)^2}+\sqrt[3]{(x^3+6x^2)(x^3-6x^2)}+\sqrt[3]{(x^3-6x^2)^2}}$$

$$=\lim_{x\to\infty}\frac{12x^2}{\sqrt[3]{x^6(1+\frac{6}{x})^2}+\sqrt[3]{x^6(1+\frac{6}{x})(1-\frac{6}{x})}+\sqrt[3]{x^6(1-\frac{6}{x})^2}}$$

$$=\lim_{x\to\infty}\frac{12x^2}{x^2[\sqrt[3]{(1+\frac{6}{x})^2}+\sqrt[3]{(1+\frac{6}{x})(1-\frac{6}{x})}+\sqrt[3]{(1-\frac{6}{x})^2}]}$$

$$=\lim_{x\to\infty}\frac{12}{\sqrt[3]{(1+\frac{6}{x})^2}+\sqrt[3]{(1+\frac{6}{x})(1-\frac{6}{x})}+\sqrt[3]{(1-\frac{6}{x})^2}}=\frac{12}{1+1+1}=4$$

ដូចនេះ: $\boxed{\lim_{x\to\infty}[\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}]=4}$

៦. របៀបគណនាលីមីតត្រង់អនន្តចំពោះកន្សោមជាផលបូកនៃស្វ៊ីតចំនួនពិត

រូបមន្តផលបូកស្វ៊ីតសំខាន់ៗ

1/ $1+2+3+......+n=\frac{n(n+1)}{2}$

2/ $1^2+2^2+3^2+.....+n^2=\frac{n(n+1)(2n+1)}{6}$

3/ $1^3+2^3+3^3+....+n^3=\frac{n^2(n+1)^2}{4}$

4/ ផលបូកស្វ៊ីតនព្វន្ត

$$S_n=a_1+a_2+a_3+....+a_n=\frac{n(a_1+a_n)}{2}$$

5/ ផលបូកស្វ៊ីតធរណីមាត្រ

$$S_n=a_1+a_2+a_3+....+a_n=a_1\times\frac{1-q^n}{1-q}$$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim\limits_{n\to+\infty}\frac{1^2+2^2+3^2+......+n^2}{n^3}$

$=\lim\limits_{n\to+\infty}\frac{n(n+1)(2n+1)}{6n^3}$

$=\lim\limits_{n\to+\infty}\frac{n^3(1+\frac{1}{n})(2+\frac{1}{n})}{6n^3}=\lim\limits_{n\to+\infty}\frac{(1+\frac{1}{n})(2+\frac{1}{n})}{6}=\frac{1.2}{6}=\frac{1}{3}$

ដូចនេះ: $\boxed{\lim\limits_{n\to+\infty}\frac{1^2+2^2+3^2+......+n^2}{n^3}=\frac{1}{3}}$

២/ $\lim\limits_{n\to+\infty}\left(\dfrac{1^3+2^3+3^3+......+n^3}{n^3}-\dfrac{n}{4}\right)$

$=\lim\limits_{n\to+\infty}\left[\dfrac{n^2(n+1)^2}{4n^3}-\dfrac{n}{4}\right]$

$=\lim\limits_{n\to+\infty}\left[\dfrac{(n+1)^2}{4n}-\dfrac{n}{4}\right]$

$=\lim\limits_{n\to+\infty}\dfrac{n^2+2n+1-n^2}{4n}$

$=\lim\limits_{n\to+\infty}\dfrac{2n+1}{4n}=\dfrac{1}{2}$

ដូចនេះ: $\boxed{\lim\limits_{n\to+\infty}\left(\dfrac{1^3+2^3+3^3+......+n^3}{n^3}-\dfrac{n}{4}\right)=\dfrac{1}{2}}$

៣/ $\lim\limits_{n\to+\infty}\left[\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+.....+\dfrac{1}{n(n+1)}\right]$

$=\lim\limits_{n\to+\infty}\left[(1-\dfrac{1}{2})+(\dfrac{1}{2}-\dfrac{1}{3})+(\dfrac{1}{3}-\dfrac{1}{4})+....+(\dfrac{1}{n}-\dfrac{1}{n+1})\right]$

$=\lim\limits_{n\to+\infty}\left(1-\dfrac{1}{n+1}\right)=1$

ដូចនេះ: $\lim\limits_{n\to+\infty}\left[\dfrac{1}{1.2}+\dfrac{1}{2.3}+\dfrac{1}{3.4}+.....+\dfrac{1}{n(n+1)}\right]=1$

៤/ $\lim_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....+\frac{1}{2^n}\right)$

$$=\lim_{n\to+\infty}\frac{1}{2}\cdot\frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}=\lim_{n\to+\infty}\left[1-\left(\frac{1}{2}\right)^n\right]=1$$

ដូចនេះ: $\lim_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....+\frac{1}{2^n}\right)=1$

៥/ $\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+.....+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)$

យើងតាង

$$S_n=\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+.....+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)$$

$$=\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left(\frac{1}{\sqrt{2k+1}+\sqrt{2k-1}}\right)$$

$$=\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left(\frac{\sqrt{2k+1}-\sqrt{2k-1}}{2k+1-2k+1}\right)=\sum_{k=1}^{n}\left(\frac{\sqrt{2k+1}-\sqrt{2k-1}}{2}\right)$$

$$=\frac{1}{\sqrt{n}}\frac{(\sqrt{3}-1)+(\sqrt{5}-\sqrt{3})+.......+(\sqrt{2n+1}-\sqrt{2n-1})}{2}$$

$$=\frac{\sqrt{2n+1}-1}{2\sqrt{n}}=\frac{1}{2}\left(\sqrt{2+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right)$$

ដូចនេះ: $\boxed{\lim_{n\to+\infty}S_n=\frac{1}{2}\lim_{n\to+\infty}\left(\sqrt{2+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right)=\frac{\sqrt{2}}{2}}$

៦/ $\lim\limits_{n\to+\infty}\left[\frac{1}{1.4.7}+\frac{1}{4.7.10}+.......+\frac{1}{(3n-2)(3n+1)(3n+4)}\right]$

$$S_n=\frac{1}{1.4.7}+\frac{1}{4.7.10}+...............+\frac{1}{(3n-2)(3n+1)(3n+4)}$$

$$=\sum_{k=1}^{n}\left[\frac{1}{(3k-2)(3k+1)(3k+4)}\right]$$

$$=\sum_{k=1}^{n}\frac{1}{6}\left[\frac{1}{(3k-2)(3k+1)}-\frac{1}{(3k+1)(3k+4)}\right]$$

$$=\frac{1}{6}\left[(\frac{1}{1.4}-\frac{1}{4.7})+...+\left(\frac{1}{(3n-2)(3n+1)}-\frac{1}{(3n+1)(3n+4)}\right)\right]$$

$$=\frac{1}{6}\left[\frac{1}{4}-\frac{1}{(3n+1)(3n+4)}\right]$$

ដូចនេះ: $\boxed{\lim\limits_{n\to+\infty}S_n=\lim\limits_{n\to+\infty}\frac{1}{6}[\frac{1}{4}-\frac{1}{(3n+1)(3n+4)}]=\frac{1}{24}}$

៧/ $\lim\limits_{n\to+\infty}(1+x)(1+x^2)(1+x^4)..........(1+x^{2^n})\quad ,|x|<1$

$$=\lim_{n\to+\infty}\left[\frac{1-x^2}{1-x}.\frac{1-x^4}{1-x^2}.\frac{1-x^8}{1-x^4}.............\frac{1-x^{2^{n+1}}}{1-x^{2^n}}\right]$$

$$=\lim_{n\to+\infty}\frac{1-x^{2^{n+1}}}{1-x}=\frac{1}{1-x}\quad ,(\lim_{n\to+\infty}x^{2^{n+1}}=0\ \forall|x|<1)$$

ដូចនេះ: $\boxed{\lim\limits_{n\to+\infty}(1+x)(1+x^2)(1+x^4)..........(1+x^{2^n})=\frac{1}{1-x}}$

៤/របៀបគណនាលីមីតអនុគមន៍ត្រីកោណមាត្រ

១. រូបមន្តអនុគមន៍ត្រីកោណមាត្រដែលគួរចងចាំ

ក-ទំនាក់ទំនងគ្រឹះ

$$1/\ \sin^2\theta+\cos^2\theta=1$$

$$2/\ \tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$3/\ \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$4/\ 1+\tan^2\theta=\frac{1}{\cos^2\theta}$$

$$5/\ 1+\cot^2\theta=\frac{1}{\sin^2\theta}$$

$$6/\ \tan\theta=\frac{1}{\cot\theta}$$

ខ-រូបមន្តបំលែងមុំផ្គុំ

1/មុំផ្ទុយគ្នា θ និង $-\theta$

$$\begin{cases}\sin(-\theta)=-\sin\theta\\ \cos(-\theta)=\cos\theta\\ \tan(-\theta)=-\tan\theta\\ \cot(-\theta)=-\cot\theta\end{cases}$$

2/ មុំបំពេញគ្នា $(\frac{\pi}{2}-\theta)$ និង θ

$$\begin{cases} \sin(\frac{\pi}{2}-\theta)=\cos\theta \\ \cos(\frac{\pi}{2}-\theta)=\sin\theta \\ \tan(\frac{\pi}{2}-\theta)=\cot\theta \\ \cot(\frac{\pi}{2}-\theta)=\tan\theta \end{cases}$$

3/ មុំបន្ថែមគ្នា $\pi-\theta$ និង θ

$$\begin{cases} \sin(\pi-\theta)=\sin\theta \\ \cos(\pi-\theta)=-\cos\theta \\ \tan(\pi-\theta)=-\tan\theta \\ \cot(\pi-\theta)=-\cot\theta \end{cases}$$

4/មុំមានផលសងស្មើនឹង π

$$\begin{cases} \sin(\pi+\theta)=-\sin\theta \\ \cos(\pi+\theta)=-\cos\theta \\ \tan(\pi+\theta)=\tan\theta \\ \cot(\pi+\theta)=\cot\theta \end{cases}$$

5/មុំមានផលសងស្មើនឹង $\frac{\pi}{2}$

$$\begin{cases} \sin(\frac{\pi}{2}+\theta)=\cos\theta \\ \cos(\frac{\pi}{2}+\theta)=-\sin\theta \\ \tan(\frac{\pi}{2}+\theta)=-\cot\theta \\ \cot(\frac{\pi}{2}+\theta)=-\tan\theta \end{cases}$$

គ-រូបមន្តផលបូកនិងផលដករវាងមុំពីរ

1/ $\cos(a-b)=\cos a\cos b+\sin a\sin b$

2/ $\cos(a+b)=\cos a\cos b-\sin a\sin b$

3/ $\sin(a+b)=\sin a\cos b+\sin b\cos a$

4/ $\sin(a-b)=\sin a\cos b-\sin b\cos a$

5/ $\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$

6/ $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

ឃ-រូបមន្តមុំឌុប

1/ $\sin 2a=\sin a\cos a$

2/ $\cos 2a=\cos^2 a-\sin^2 a=2\cos^2 a-1=1-2\sin^2 a$

3/ $\tan 2a = \dfrac{2\tan a}{1-\tan^2 a}$

ង-រូបមន្តបំលែងពីផលគុណទៅផលបូក

1/ $\sin a\cos b = \dfrac{1}{2}[\sin(a+b)+\sin(a-b)]$

2/ $\sin b\cos a = \dfrac{1}{2}[\sin(a+b)-\sin(a-b)]$

3/ $\cos a\cos b = \dfrac{1}{2}[\cos(a+b)+\cos(a-b)]$

4/ $\sin a\sin b = -\dfrac{1}{2}[\cos(a+b)-\cos(a-b)]$

ច-រូបមន្តបំលែងពីផលគុណទៅផលបូក

1/ $\sin p+\sin q = 2\sin\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$

2/ $\sin p-\sin q = 2\sin\dfrac{p-q}{2}\cos\dfrac{p+q}{2}$

3/ $\cos p+\cos q = 2\cos\dfrac{p+q}{2}\cos\dfrac{p-q}{2}$

4/ $\cos p-\cos q = -2\sin\dfrac{p-q}{2}\sin\dfrac{p+q}{2}$

5/ $\tan p+\tan q = \dfrac{\sin(p+q)}{\cos p\cos q}$

6/ $\tan p-\tan q = \dfrac{\sin(p-q)}{\cos p\cos q}$

7/ $\cot p+\cot q=\dfrac{\sin(p+q)}{\sin p\sin q}$

8/ $\cot p-\cot q=-\dfrac{\sin(p-q)}{\sin p\sin q}$

ឆ-កន្សោម $\sin 3a$, $\cos 3a$ និង $\tan 3a$

1/ $\sin 3a=3\sin a-4\sin^3 a$

2/ $\cos 3a=4\cos^3 a-3\cos a$

3/ $\tan 3a=\dfrac{3\tan a-\tan^3 a}{1-3\tan^2 a}$

ជ/ អនុគមន៍ត្រីកោណមាត្រនៃមុំ $\theta+2k\pi$ និង θ

1/ $\sin(\theta+2k\pi)=\sin\theta$

2/ $\cos(\theta+2k\pi)=\cos\theta$

3/ $\tan(\theta+2k\pi)=\tan\theta$

4/ $\cot(\theta+2k\pi)=\cot\theta$

(ដែល k ជាចំនួនគត់រ៉ឺឡាទីហ្វ) ។

៦. របៀបគណនាដោយប្រើរូបមន្តគ្រឹះ

1/ $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=\lim\limits_{x\to 0}\dfrac{x}{\sin x}=1$

2/ $\lim\limits_{x\to 0}\dfrac{\tan x}{x}=\lim\limits_{x\to 0}\dfrac{x}{\tan x}=1$

3/ $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=0$

ឧទាហរណ៍គំរូ

គណនាលីមីតអនុគមន៍ត្រីកោណមាត្រខាងក្រោម

1/ $\lim_{x\to 0}\frac{\sin x\sin 2x\sin 3x}{x^3}$

$=\lim_{x\to 0}\frac{\sin x}{x}.\frac{\sin 2x}{x}.\frac{\sin 3x}{x}$

$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right).\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right).\lim_{x\to 0}\left(3.\frac{\sin 3x}{3x}\right)$

$=1\times 2\times 3=6$

ដូចនេះ: $\lim_{x\to 0}\frac{\sin x\sin 2x\sin 3x}{x^3}=6$

2/ $\lim_{x\to 0}\frac{\sin x+\sin 2x+\sin 3x}{x}$

$=\lim_{x\to 0}\left(\frac{\sin x}{x}+\frac{\sin 2x}{x}+\frac{\sin 3x}{x}\right)$

$=\lim_{x\to 0}\left(\frac{\sin x}{x}\right)+\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right)+\lim_{x\to 0}\left(3.\frac{\sin 3x}{3x}\right)$

$=1+2+3=6$

ដូចនេះ: $\lim_{x\to 0}\frac{\sin x+\sin 2x+\sin 3x}{x}=6$

3/ $\lim_{x\to 0}\frac{x+\sin 3x}{x}$

$=\lim_{x\to 0}\left(1+3.\frac{\sin 3x}{3x}\right)=1+3=4$

4/ $\lim_{x\to 0}\frac{\sin 2x}{\sin 6x}$

$$=\lim_{x\to 0}\left(\frac{\sin 2x}{x}.\frac{x}{\sin 6x}\right)$$

$$=\lim_{x\to 0}\left(2.\frac{\sin 2x}{2x}\right).\lim_{x\to 0}\left(\frac{1}{6}.\frac{6x}{\sin 6x}\right)$$

$$=2\times\frac{1}{6}=\frac{1}{3}$$

ដូចនេះ: $\lim_{x\to 0}\frac{\sin 2x}{\sin 6x}=\frac{1}{3}$

5/ $\lim_{x\to 0}\frac{x+\sin 3x}{x+\tan x}$

$$=\lim_{x\to 0}\frac{x\left(1+\frac{\sin 3x}{x}\right)}{x\left(1+\frac{\tan x}{x}\right)}$$

$$=\lim_{x\to 0}\frac{1+3.\frac{\sin 3x}{3x}}{1+\frac{\tan x}{x}}=\frac{1+3}{1+1}=2$$

ដូចនេះ: $\lim_{x\to 0}\frac{x+\sin 3x}{x+\tan x}=2$

7/ $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$

$=\lim\limits_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}$

$=2\lim\limits_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\times\frac{1}{4}=\frac{1}{2}$

ដូចនេះ: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

8/ $\lim\limits_{x\to 0}\frac{1-\cos 2x}{1-\cos 4x}$

$=\lim\limits_{x\to 0}\frac{2\sin^2 x}{2\sin^2 2x}$

$=\lim\limits_{x\to 0}\left(\frac{\sin^2 x}{x^2}.\frac{x^2}{\sin^2 2x}\right)$

$=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^2\lim\limits_{x\to 0}\left(\frac{2x}{\sin 2x}\right)^2\times\frac{1}{4}=\frac{1}{4}$

ដូចនេះ: $\lim\limits_{x\to 0}\frac{1-\cos 2x}{1-\cos 4x}=\frac{1}{4}$

9/ $\lim_{x\to 0}\frac{1-\cos^3 2x}{x\sin x}$

$$=\lim_{x\to 0}\frac{(1-\cos 2x)(1+\cos 2x+\cos^2 2x)}{x\sin x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x(1+\cos 2x+\cos^2 2x)}{x\sin x}$$

$$=2\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}(1+\cos 2x+\cos^2 2x)$$

$$=2.1.3=6$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{1-\cos^3 2x}{x\sin x}=6}$ ។

10/ $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$

ដោយ $\tan x=\frac{\sin x}{\cos x}\Rightarrow \sin x=\cos x.\tan x$

$$=\lim_{x\to 0}\frac{\tan x-\cos x.\tan x}{x^3}$$

$$=\lim_{x\to 0}\frac{\tan x(1-\cos x)}{x^3}$$

$$=\lim_{x\to 0}\frac{2\tan x\sin^2\frac{x}{2}}{x^3}$$

$$=2\lim_{x\to 0}\frac{\tan x}{x}.\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}=2.1.\frac{1}{4}=\frac{1}{2}$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}}$ ។

11/ $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}$

$$=\lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{3\sin x-(3\sin x-4\sin^3 x)}$$

$$=\lim_{x\to 0}\frac{2\sin x(1-\cos x)}{4\sin^3 x}$$

$$=\lim_{x\to 0}\frac{4\sin x\sin^2\frac{x}{2}}{4\sin^3 x}=\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\sin^2 x}$$

$$=\lim_{x\to 0}\left[\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2.\frac{1}{4}.\frac{x^2}{\sin^2 x}\right]=1^2.\frac{1}{4}.1^2=\frac{1}{4}$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{2\sin x-\sin 2x}{3\sin x-\sin 3x}=\frac{1}{4}}$ ។

12/ $\lim_{x\to 0}\frac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}$

$$=\lim_{x\to 0}\frac{(1-\cos 4x)-(1-\cos 2x)}{(1-\cos 2x)+\cos 2x(1-\cos 4x)}$$

$$=\lim_{x\to 0}\frac{2\sin^2 2x-2\sin^2 x}{2\sin^2 x+2\cos 2x\sin^2 2x}$$

$$=\lim_{x\to 0}\frac{(1-\cos 4x)-(1-\cos 2x)}{(1-\cos 2x)+\cos 2x(1-\cos 4x)}$$

$$=\lim_{x\to 0}\frac{2\sin^2 2x-2\sin^2 x}{2\sin^2 x+2\cos 2x\sin^2 2x}$$

$$=\lim_{x\to 0}\frac{\sin^2 2x-\sin^2 x}{\sin^2 x+\cos 2x\sin^2 2x}$$

$$=\lim_{x\to 0}\frac{\frac{\sin^2 2x}{x^2}-\frac{\sin^2 x}{x^2}}{\frac{\sin^2 x}{x^2}+\cos 2x.\frac{\sin^2 2x}{x^2}}=\frac{4-1}{1+4}=\frac{3}{5}$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{\cos 2x-\cos 4x}{1-\cos 2x\cos 4x}=\frac{3}{5}}$

13/ $\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$

$$=\lim_{x\to 0}\frac{1-\cos(2\sin^2\frac{x}{2})}{x^4}=\lim_{x\to 0}\frac{2\sin^2(\sin^2\frac{x}{2})}{x^4}$$

$$=2\lim_{x\to 0}\frac{\sin^2(\sin^2\frac{x}{2})}{(\sin^2\frac{x}{2})^2}.\frac{\sin^4\frac{x}{2}}{(\frac{x}{2})^4}.\frac{1}{16}=2.\frac{1}{16}=\frac{1}{8}$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\frac{1}{8}}$ ។

14/ $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$=\lim_{x\to 0}\frac{(1-\cos x)+\cos x(1-\sqrt{\cos 2x})}{x^2}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}+\lim_{x\to 0}\frac{\cos x(1-\cos 2x)}{x^2(1+\sqrt{\cos 2x})}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}+\lim_{x\to 0}\frac{2\cos x\sin^2 x}{x^2(1+\sqrt{\cos 2x})}$$

$$=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}.\frac{1}{4}+2\lim_{x\to 0}\frac{\sin^2 x}{x^2}.\frac{\cos x}{1+\sqrt{\cos 2x}}$$

$$=2.\frac{1}{2}+2.\frac{1}{2}=2$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=2}$ ។

15/ $\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}$

$$=\lim_{x\to 0}\frac{1-\cos x}{1-\cos 2x}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin^2 x}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2\sin^2 x}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}$$

$$=\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}.\frac{x^2}{\sin^2 x}.\frac{1}{4}.\frac{1+\sqrt{\cos 2x}}{1+\sqrt{\cos x}}=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}=\frac{1}{4}}$ ។

16/ $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}$

$$=\lim_{x\to 0}\frac{2-1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$=\lim_{x\to 0}\frac{1-\cos x}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x\tan x(\sqrt{2}+\sqrt{1+\cos x})}$$

$$=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}.\frac{x}{\tan x}.\frac{1}{4}.\frac{1}{\sqrt{2}+\sqrt{1+\cos x}}=\frac{\sqrt{2}}{8}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{x.\tan x}=\frac{\sqrt{2}}{8}}$ ។

17/ $\lim_{x\to 0}\frac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}$

$$=\lim_{x\to 0}\frac{1+\sin^2 x-\cos^2 x}{\cos x-\cos 3x}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=\lim_{x\to 0}\frac{\sin^2 x+(1-\cos^2 x)}{2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=\lim_{x\to 0}\frac{2\sin^2 x}{2\sin 2x.\sin(-x)}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}$$

$$=-\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{2x}{\sin 2x}\cdot\frac{1}{2}\cdot\frac{\sqrt{\cos x}+\sqrt{\cos 3x}}{\sqrt{1+\sin^2 x}+\cos x}=-\frac{1}{2}$$

ដូចនេះ $\lim_{x\to 0}\frac{\sqrt{1+\sin^2}-\cos x}{\sqrt{\cos x-\sqrt{\cos 3x}}}=-\frac{1}{2}$

៦. របៀបគណនាត្រីកោណមាត្រដោយប្រើរូបមន្តប្តូរអថេរ

ឧបមាថាគេមានលីមីត $L=\lim_{x\to a}[f(x)]$ ដែល $a\neq 0$

គេតាង $t=a-x\Rightarrow x=a-t$

កាលណា $x\to a$ នោះ $t\to 0$

គេបានរូបមន្ត $L=\lim_{x\to a}[f(x)]=\lim_{t\to 0}[f(a-t)]$

(ហៅថារូបមន្តប្តូរអថេរ) ។

ឧទាហរណ៍ ចូរគណនាលីមីតខាងក្រោម

1/ $\lim\limits_{x\to 1}\dfrac{1-\sin\frac{\pi x}{2}}{(1-x)^2}$

តាង $z=1-x$ នាំឲ្យ $x=1-z$

កាលណា $x\to 1$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{1-\sin(\frac{\pi}{2}-\frac{\pi z}{2})}{z^2}=\lim_{z\to 0}\frac{1-\cos\frac{\pi z}{2}}{z^2}$$

$$=\lim_{z\to 0}\frac{2\sin^2\frac{\pi z}{4}}{z^2}=2\lim_{z\to 0}\frac{\sin^2\frac{\pi z}{4}}{(\frac{\pi z}{4})^2}.\frac{\pi^2}{16}=\frac{\pi^2}{8}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}\dfrac{1-\sin\frac{\pi x}{2}}{(1-x)^2}=\dfrac{\pi^2}{8}}$ ។

2/ $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{\pi^2-4x^2}$

តាង $z=\frac{\pi}{2}-x$ នាំឲ្យ $x=\frac{\pi}{2}-z$

កាលណា $x\to\frac{\pi}{2}$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{\cos(\frac{\pi}{2}-z)}{\pi^2-4(\frac{\pi}{2}-z)^2}$$

$$=\lim_{z\to 0}\frac{\sin z}{\pi^2-\pi^2+4\pi z-4z^2}$$

$$=\lim_{z\to 0}\frac{\sin z}{4\pi z-4z^2}=\lim_{z\to 0}\frac{\sin z}{z}.\frac{1}{4\pi-4z}=\frac{1}{4\pi}$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{2}}\frac{\cos x}{\pi^2-4x^2}=\frac{1}{4\pi}}$ ។

3/ $\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\pi-4x}$

តាង $z=\frac{\pi}{4}-x$ នាំឲ្យ $x=\frac{\pi}{4}-z$

កាលណា $x\to\frac{\pi}{4}$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{\sin(\frac{\pi}{4}-z)-\cos(\frac{\pi}{4}-z)}{\pi-4(\frac{\pi}{4}-z)}$$

$$=\lim_{z\to 0}\frac{(\frac{\sqrt{2}}{2}\cos z-\frac{\sqrt{2}}{2}\sin z)-(\frac{\sqrt{2}}{2}\cos z+\frac{\sqrt{2}}{2}\sin z)}{4z}$$

$$=\lim_{z\to 0}\frac{-\sqrt{2}\sin z}{4z}$$

$$=-\frac{\sqrt{2}}{4}\lim_{z\to 0}\frac{\sin z}{z}=-\frac{\sqrt{2}}{4}$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\pi-4x}=-\frac{\sqrt{2}}{4}}$

4/ $\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$

តាង $z=\frac{\pi}{2}-x$ នាំឲ្យ $x=\frac{\pi}{2}-z$

កាលណា $x\to\frac{\pi}{2}$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{\sqrt{2}-\sqrt{1+\sin(\frac{\pi}{2}-z)}}{\cos^2(\frac{\pi}{2}-z)}$$

$$=\lim_{z\to 0}\frac{\sqrt{2}-\sqrt{1+\cos z}}{\sin^2 z}$$

$$=\lim_{z\to 0}\frac{2-1-\cos z}{\sin^2 z.(\sqrt{2}+\sqrt{1+\cos z})}$$

$$=\lim_{z\to 0}\frac{1-\cos z}{\sin^2 z(\sqrt{2}+\sqrt{1+\cos z})}$$

$$=\lim_{z\to 0}\frac{2-1-\cos z}{\sin^2 z.(\sqrt{2}+\sqrt{1+\cos z})}$$

$$=\lim_{z\to 0}\frac{1-\cos z}{\sin^2 z(\sqrt{2}+\sqrt{1+\cos z})}$$

$$=\lim_{z\to 0}\frac{2\sin^2\frac{z}{2}}{\sin^2 z.(\sqrt{2}+\sqrt{1+\cos z})}$$

$$=2\lim_{z\to 0}\frac{\sin^2\frac{z}{2}}{(\frac{z}{2})^2}.\frac{z^2}{\sin^2 z}.\frac{1}{4}.\frac{1}{\sqrt{2}+\sqrt{1+\cos z}}$$

$$=2.\frac{1}{4}.\frac{1}{2\sqrt{2}}=\frac{\sqrt{2}}{8}$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}=\frac{\sqrt{2}}{8}}$ ។

5/ $\lim_{x\to 1}\frac{x^3-1+\tan\pi x}{1-x^2}$

តាង $z=1-x$ នាំឲ្យ $x=1-z$

កាលណា $x\to 1$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{(1-z)^3-1+\tan(\pi-\pi z)}{1-(1-z)^2}$$

$$=\lim_{z\to 0}\frac{(1-z)^3-1+\tan(\pi-\pi z)}{1-(1-z)^2}$$

$$=\lim_{z\to 0}\frac{1-3z+3z^2-z^3-1-\tan\pi z}{1-1+2z-z^2}$$

$$=\lim_{z\to 0}\frac{-3z+3z^2-z^3-\tan\pi z}{2z-z^2}$$

$$=\lim_{z\to 0}\frac{z(-3+3z-z^2-\frac{\tan\pi z}{z})}{z(2-z)}=\frac{-3-\pi}{2}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x^3-1+\tan\pi x}{1-x^2}=-\frac{3+\pi}{2}}$

6/ $\lim_{x\to 2}(4-x^2)\tan\frac{\pi x}{4}$

តាង $z=2-x$ នោះ $x=2-z$

កាលណា $x\to 2$ នោះ $z\to 0$

$$=\lim_{z\to 0}\left[4-(2-z)^2\right]\tan\frac{\pi}{4}(2-z)$$

$$=\lim_{z\to 0}(4z-z^2)\tan(\frac{\pi}{2}-\frac{\pi z}{4})$$

$$=\lim_{z\to 0}(4-z)\frac{\frac{\pi z}{4}}{\tan\frac{\pi z}{4}}.\frac{4}{\pi}=\frac{16}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to 2}(4-x^2)\tan\frac{\pi x}{4}=\frac{16}{\pi}}$ ។

7/ $\lim\limits_{x\to\pi}(\pi-x)\tan\frac{x}{2}$

តាង $z=\pi-x\Rightarrow x=\pi-z$

កាលណា $x\to\pi$ នោះ $z\to0$

$$=\lim_{z\to0} z\tan\frac{\pi-z}{2}=\lim_{z\to0} z\tan(\frac{\pi}{2}-\frac{z}{2})$$

$$=\lim_{z\to0} z\cot z=\lim_{z\to0}\frac{z}{\tan z}=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to\pi}(\pi-x)\tan\frac{x}{2}=1}$

8/ $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$

តាង $z=\frac{\pi}{4}-x\Rightarrow x=\frac{\pi}{4}-z$

កាលណា $x\to\frac{\pi}{4}$ នោះ $z\to0$

$$=\lim_{z\to0}\frac{1-\tan(\frac{\pi}{4}-z)}{1-\sqrt{2}\sin(\frac{\pi}{4}-z)}$$

$$=\lim_{z\to0}\frac{1-\frac{1-\tan z}{1+\tan z}}{1-\sqrt{2}(\frac{\sqrt{2}}{2}\cos z-\frac{\sqrt{2}}{2}\sin z)}$$

$$= \lim_{z\to 0} \frac{1-\tan(\frac{\pi}{4}-z)}{1-\sqrt{2}\sin(\frac{\pi}{4}-z)}$$

$$= \lim_{z\to 0} \frac{1-\frac{1-\tan z}{1+\tan z}}{1-\sqrt{2}(\frac{\sqrt{2}}{2}\cos z-\frac{\sqrt{2}}{2}\sin z)}$$

$$= \lim_{z\to 0} \frac{2\tan z}{(1-\cos z+\sin z)(1+\tan z)}$$

$$= 2\lim_{z\to 0} \frac{\frac{\tan z}{z}}{(\frac{1-\cos z}{z}+\frac{\sin z}{z})(1+\tan z)}$$

$$= 2.\frac{1}{(0+1)(1+0)} = 2$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{1-\sqrt{2}\sin x} = 2}$ ។

9/ $\lim_{x\to\frac{\pi}{3}} \frac{\pi-3x}{\sqrt{3}-2\sin x}$

តាង $z = \frac{\pi}{3} - x \Rightarrow x = \frac{\pi}{3} - z$

កាលណា $x \to \frac{\pi}{3}$ នោះ $z \to 0$

$$=\lim_{z\to 0}\frac{\pi-3(\frac{\pi}{3}-z)}{\sqrt{3}-2\sin(\frac{\pi}{3}-z)}$$

$$=\lim_{z\to 0}\frac{3z}{\sqrt{3}-2(\frac{\sqrt{3}}{2}\cos z-\frac{1}{2}\sin z)}$$

$$=\lim_{z\to 0}\frac{3z}{\sqrt{3}-\sqrt{3}\cos z-\sin z}$$

$$=\lim_{z\to 0}\frac{3}{\sqrt{3}\frac{1-\cos z}{z}-\frac{\sin z}{z}}=\frac{3}{0-1}=-3$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{3}}\frac{\pi-3x}{\sqrt{3}-2\sin x}=-3}$ ។

10/ $\lim_{x\to\pi}\frac{\cos\frac{\pi x}{x+\pi}}{\pi-x}$

តាង $t=\frac{\pi x}{x+\pi}\Rightarrow x=\frac{\pi t}{\pi-t}$ បើ $x\to\pi\ \Rightarrow t\to\frac{\pi}{2}$

$$=\lim_{t\to\frac{\pi}{2}}\frac{\cos t}{\pi-\frac{\pi t}{\pi-t}}$$

$$=\lim_{t\to\frac{\pi}{2}}\frac{(\pi-t)\cos t}{\pi(\pi-2t)}$$

តាង $u=\frac{\pi}{2}-t \Rightarrow t=\frac{\pi}{2}-u$ បើ $t\to\frac{\pi}{2}\Rightarrow u\to 0$

$$=\lim_{u\to 0}\frac{(\pi-\frac{\pi}{2}+u)\cos(\frac{\pi}{2}-u)}{\pi(\pi-\pi+2u)}$$

$$=\lim_{u\to 0}\frac{\frac{\pi}{2}+u}{2\pi}.\frac{\sin u}{u}=\frac{1}{4}$$

ដូចនេះ $\lim_{x\to\pi}\frac{\cos\frac{\pi x}{x+\pi}}{\pi-x}=\frac{1}{4}$ ។

11/ $\lim_{x\to 2}\frac{2-x}{\sin\frac{2\pi}{x}}$

តាង $z=\frac{2}{x}\Rightarrow x=\frac{2}{z}$ បើ $x\to 2$ នោះ $z\to 1$

$$=\lim_{z\to 1}\frac{2-\frac{2}{z}}{\sin\pi z}=2\lim_{z\to 1}\frac{z-1}{z.\sin\pi z}$$

តាង $z=1-u\Rightarrow u=1-z$ បើ $z\to 1$ នោះ $u\to 0$

$$=2\lim_{u\to 0}\frac{-u}{(1-u)\sin(\pi-\pi u)}$$

$$=-2\lim_{u\to 0}\frac{1}{1-u}.\frac{\pi u}{\sin\pi u}.\frac{1}{\pi}=-\frac{2}{\pi}$$

៥/លីមីតនៃអនុគមន៍អិចស្ប៉ូណង់ស្យែល

ក/ $\lim_{x\to 0}\frac{e^x-1}{x}=1$

ខ/ $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a \quad ,(a>0)$

គ/ $\lim_{x\to 0}\left(1+x\right)^{\frac{1}{x}}=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

ឃ/ $\lim_{x\to+\infty} a^x = \begin{cases} +\infty & \text{បើ } a>1 \\ 0 & \text{បើ } 0<a<1 \end{cases}$

ឧទាហរណ៍ ចូរគណនាលីមីតខាងក្រោម

1/ $\lim_{x\to 0}\frac{e^{3x}+2x-1}{x}$

$=\lim_{x\to 0}(\frac{e^{3x}-1}{x}+2)$

$=\lim_{x\to 0}(3.\frac{e^{3x}-1}{3x})+\lim_{x\to 0}(2)$

$=3+2=5$

ដូចនេះ $\lim_{x\to 0}\frac{e^{3x}+2x-1}{x}=5$

2/ $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{x}$

$=\lim\limits_{x\to 0}\dfrac{(e^{2x}-1)-(e^{-2x}-1)}{x}$

$=2\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{2x}+2\lim\limits_{x\to 0}\dfrac{e^{-2x}-1}{-2x}$

$=2+2=4$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{x}=4$

3/ $\lim\limits_{x\to 0}\dfrac{e^{x}+e^{-x}-2}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{e^{x}+\dfrac{1}{e^{x}}-2}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{e^{2x}-2e^{x}+1}{x^2}$

$=\lim\limits_{x\to 0}\left(\dfrac{e^{x}-1}{x}\right)^2=1$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^{x}+e^{-x}-2}{x^2}=1$

4/ $\lim\limits_{x\to 0}\dfrac{2e^{2x}-3e^{x}+1}{x}$

$=\lim\limits_{x\to 0}\dfrac{2e^{x}(e^{2x}-1)-(e^{x}-1)}{x}$

$$= \lim_{x\to 0}\left(2e^x.\frac{e^{2x}-1}{2x}\times 2\right) - \lim_{x\to 0}\frac{e^x-1}{x}$$

$$= 2\times 2 - 1 = 3$$

ដូចនេះ: $\lim_{x\to 0}\frac{2e^{2x}-3e^x+1}{x} = 3$

5/ $\lim_{x\to 0}\frac{e^x - e^{-x}}{\sin 2x}$

$$= \lim_{x\to 0}\frac{e^x - \frac{1}{e^x}}{\sin 2x} = \lim_{x\to 0}\frac{e^{2x}-1}{e^x \sin 2x}$$

$$= \lim_{x\to 0}\frac{e^{2x}-1}{2x}.\frac{2x}{\sin 2x}.\frac{1}{e^x} = 1$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{e^x - e^{-x}}{\sin 2x} = 1}$ ។

6/ $\lim_{x\to 0}\frac{e^{ax} - e^{bx}}{x}$ $,a,b \in IR^*$

$$= \lim_{x\to 0}\frac{(e^{ax}-1)-(e^{bx}-1)}{x}$$

$$= \lim_{x\to 0}\frac{e^{ax}-1}{ax}.a - \lim_{x\to 0}\frac{e^{bx}-1}{bx}.b = a - b$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x} = a - b}$

7/ $\lim_{x\to 0}\frac{2e^x+3e^{2x}-5}{e^{3x}-1}$

$=\lim_{x\to 0}\frac{2(e^x-1)+3(e^{2x}-1)}{(e^{3x}-1)}$

$=\lim_{x\to 0}\frac{2.\frac{e^x-1}{x}+6.\frac{e^{2x}-1}{2x}}{3.\frac{e^{3x}-1}{3x}}=\frac{2+6}{3}=\frac{8}{3}$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{2e^x+3e^{2x}-5}{e^{3x}-1}=\frac{8}{3}}$ ។

8/ $\lim_{x\to 0}\frac{e^x+e^{2x}+...+e^{nx}-n}{x}$

$=\lim_{x\to 0}\frac{(e^x-1)+(e^{2x}-1)+........+(e^{nx}-1)}{x}$

$=\lim_{x\to 0}\left[\frac{e^x-1}{x}+2.\frac{e^{2x}-1}{2x}+.....+n.\frac{e^{nx}-1}{nx}\right]$

$=1+2+..........+n=\frac{n(n+1)}{2}$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^x+e^{2x}+...+e^{nx}-n}{x}=\frac{n(n+1)}{2}}$ ។

$$9/\lim_{x\to 0}\frac{(e^{x}-1)(e^{2x}-1)...(e^{nx}-1)}{x^{n}}$$

$$=\lim_{x\to 0}\frac{e^{x}-1}{x}.2\frac{e^{2x}-1}{2x}......n\frac{e^{nx}-1}{nx}=1.2.3......n=n!$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{(e^{x}-1)(e^{2x}-1)...(e^{nx}-1)}{x^{n}}=\frac{n(n+1)}{2}}$ ។

$$10/\lim_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{-x^2}-(1-2\sin^2 x)}{x^2}$$

$$=\lim_{x\to 0}\frac{e^{-x^2}-1+2\sin^2 x}{x^2}$$

$$=-\lim_{x\to 0}\frac{e^{-x^2}-1}{-x^2}+2\lim_{x\to 0}\frac{\sin^2 x}{x}=-1+2=1$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}=1}$ ។

$$11/\lim_{x\to 0}\frac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}$$

$$=\lim_{x\to 0}\frac{(e^{-2\sin x}-1)-(e^{\tan 3x}-1)}{x(x^2+1)}$$

$$=\lim_{x\to 0}\frac{e^{-2\sin x}-1}{-2\sin x}\cdot\frac{\sin x}{x}\cdot\frac{-2}{x^2+1}-\lim_{x\to 0}\frac{e^{\tan 3x}-1}{\tan 3x}\cdot\frac{\tan 3x}{3x}\cdot\frac{3}{x^2+1}$$

$$=-2-3=-5$$

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}=-5}$

12/ $\lim_{x\to 0}\frac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}$

$$=\lim_{x\to 0}\frac{x(e^{-2x^2}-1)+\cos x\tan x-\tan x}{x^3}$$

$$=\lim_{x\to 0}\frac{e^{-2x^2}-1}{x^2}+\lim_{x\to 0}\frac{\tan x(\cos x-1)}{x^3}$$

$$=-2\lim_{x\to 0}\frac{e^{-2x^2}-1}{-2x^2}-2\lim_{x\to 0}\frac{\tan x}{x}\cdot\frac{\sin^2\frac{x}{2}}{x^2}$$

$$=-2-2(\frac{1}{2})^2=-\frac{5}{2}$$

ដូចនេះ: $\lim_{x\to 0}\frac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}=-\frac{5}{2}$

13/ $\lim_{x\to 0}\frac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}$

$$=\lim_{x\to 0}\frac{e^{3\sin^2 x}-(1-2\sin^2\frac{x}{2})(1-2\sin^2\frac{3x}{2})}{-e^{2x^2}+1-2\sin^2 x}$$

$$=\lim_{x\to0}\frac{e^{3\sin^2 x}-(1-2\sin^2\frac{x}{2})(1-2\sin^2\frac{3x}{2})}{-e^{2x^2}+1-2\sin^2 x}$$

$$=\lim_{x\to0}\frac{e^{3\sin^2 x}-1+2\sin^2\frac{3x}{2}+2\sin^2\frac{x}{2}-4\sin^2\frac{x}{2}\sin^2\frac{3x}{2}}{-(e^{2x^2}-1+2\sin^2 x)}$$

$$=-\lim_{x\to0}\frac{3.\frac{e^{3\sin^2 x}-1}{3\sin^2 x}.\frac{\sin^2 x}{x^2}+2\frac{\sin^2\frac{3x}{2}}{x^2}+2\frac{\sin^2\frac{x}{2}}{x^2}-4\frac{\sin^2\frac{x}{2}}{x^2}.\sin^2\frac{3x}{2}}{2.\frac{e^{2x^2}-1}{2x^2}+2.\frac{\sin^2 x}{x^2}}$$

$$=-\frac{3+2.\frac{9}{4}-2.\frac{1}{4}-0}{2+2}=\frac{3+4}{4}=\frac{7}{4}$$

ដូចនេះ $\lim_{x\to0}\frac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}=\frac{7}{4}$

14/ $\lim_{x\to0}\frac{4^x+2^x-2}{x}$

$$=\lim_{x\to0}\frac{(4^x-1)+(2^x-1)}{x}$$

$$=\lim_{x\to0}\frac{4^x-1}{x}+\lim_{x\to0}\frac{2^x-1}{x}$$

$$=\ln 4-\ln 2=\ln\left(\frac{4}{2}\right)=\ln 2$$

ដូចនេះ $\lim_{x\to0}\frac{4^x+2^x-2}{x}=\ln 2$

15/ $\lim\limits_{x\to 0}\frac{7^{x^2}-\cos 2x}{x^2}$

$$=\lim_{x\to 0}\frac{(7^{x^2}-1)+(1-\cos 2x)}{x^2}$$

$$=\lim_{x\to 0}\frac{7^{x^2}-1}{x^2}+\lim_{x\to 0}\frac{2\sin^2 x}{x^2}$$

$$=\ln 7+2$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{7^{x^2}-\cos 2x}{x^2}=2+\ln 7$

16/ $\lim\limits_{x\to 0}\frac{2^{\sin x}-3^{\tan x}}{x}$

$$=\lim_{x\to 0}\frac{(2^{\sin x}-1)-(3^{\tan x}-1)}{x}$$

$$=\lim_{x\to 0}\frac{2^{\sin x}-1}{\sin x}\times\frac{\sin x}{x}-\lim_{x\to 0}\frac{3^{\tan x}-1}{\tan x}\times\frac{\tan x}{x}$$

$$=\ln 2-\ln 3=\ln\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{2^{\sin x}-3^{\tan x}}{x}=\ln\frac{2}{3}$

៦/លីមីតនៃអនុគមន៍លោការីត

ក/ $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$

ខ/ $\lim\limits_{x\to 0^{+}}\ln x=-\infty$

គ/ $\lim\limits_{x\to +\infty}\ln x=+\infty$

ឧទាហរណ៍ ចូរគណនាលីមីតខាងក្រោម

1/ $\lim\limits_{x\to 0}\frac{\ln(1+2x^{2})}{1-\cos 2x}$

$=\lim\limits_{x\to 0}\frac{\ln(1+2x^{2})}{2\sin^{2}x}$

$=\lim\limits_{x\to 0}\frac{\ln(1+2x^{2})}{2x^{2}}\cdot\frac{x^{2}}{\sin^{2}x}=1$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\ln(1+2x^{2})}{1-\cos 2x}=1$

2/ $\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^{2}}$

$=\lim\limits_{x\to 0}\frac{\ln(1-2\sin^{2}x)}{x^{2}}$

$=\lim\limits_{x\to 0}\frac{\ln(1-2\sin^{2}x)}{-2\sin^{2}x}\times\frac{-2\sin^{2}x}{x^{2}}=-2$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^{2}}=-2$

$$3/\lim_{x\to 0}\frac{\ln(x^2-x+1)}{x}$$

$$=\lim_{x\to 0}\frac{\ln[1+x(x-1)]}{x(x-1)}\times(x-1)=-1$$

ដូចនេះ: $\lim_{x\to 0}\frac{\ln(x^2-x+1)}{x}=-1$

$$4/\lim_{x\to 0}\frac{\ln(1+x)+\ln(1+2x)+\ln(1+3x)}{x}$$

$$=\lim_{x\to 0}\left[\frac{\ln(1+x)}{x}+2\frac{\ln(1+2x)}{2x}+3\frac{\ln(1+3x)}{3x}\right]$$

$$=1+2+3=6$$

ដូចនេះ: $\lim_{x\to 0}\frac{\ln(1+x)+\ln(1+2x)+\ln(1+3x)}{x}=6$

$$5/\lim_{x\to 0}\frac{\ln(1+2\sin^2 x)}{3\tan^2 x}$$

$$=\lim_{x\to 0}\frac{\ln(1+2\sin^2 x)}{2\sin^2 x}\times\frac{2\sin^2 x}{x^2}\times\frac{x^2}{3\tan^2 x}$$

$$=\frac{2}{3}\lim_{x\to 0}\frac{\ln(1+2\sin^2 x)}{2\sin^2 x}\times\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2\times\lim_{x\to 0}\left(\frac{x}{\tan x}\right)^2$$

$$=\frac{2}{3}$$

ដូចនេះ: $\lim_{x\to 0}\frac{\ln(1+2\sin^2 x)}{3\tan^2 x}=\frac{2}{3}$ ។

៧/របៀបគណនាលីមីតរាងមិនកំនត់ 1^{∞}

កាលណា $x \to x_0$ គេមាន $f(x) \to 0$ និង $g(x) \to \infty$

នោះលីមីត $\lim_{x\to x_0} [f(x)]^{g(x)}$ មានរាងមិនកំនត់ 1^{∞} ។

ដើម្បីគណនាលីមីតនេះគេត្រូវប្រើរូបមន្តគ្រឹះ

$$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e \quad \text{ឬ} \quad \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} \text{ ។}$$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim_{x\to 0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}$

គេមាន $\frac{x^2-x+1}{x^2+x+1} = 1+\left(\frac{x^2-x+1}{x^2+x+1}-1\right) = 1+\frac{-2x}{x^2+x+1}$

$$\lim_{x\to 0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}} = \lim_{x\to 0}\left(1+\frac{-2x}{x^2+x+1}\right)^{\frac{1}{\sin x}}$$

$$= \lim_{x\to 0}\left[\left(1+\frac{-2x}{x^2+x+1}\right)^{\frac{x^2+x+1}{-2x}}\right]^{-\frac{x}{\sin x}\cdot\frac{2}{x^2+x+1}} = e^{-2} = \frac{1}{e^2}$$

ដូចនេះ $\lim_{x\to 0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}=\frac{1}{e^2}$

២/ $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$

$$=\lim_{x\to 0}\left[1+(\frac{\sin x-x}{x})\right]^{\frac{x}{\sin x-x}\times\frac{-\tan x}{x}}=e^{-1}=\frac{1}{e}$$

ដូចនេះ $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}=\frac{1}{e}$

៣/ $\lim_{x\to 0}(2-\cos 2x)^{\frac{1}{x^2}}$

$$=\lim_{x\to 0}[1+(1-\cos 2x)]^{\frac{1}{1-\cos 2x}\times\frac{2\sin^2 x}{x^2}}=e^2$$

ដូចនេះ $\lim_{x\to 0}(2-\cos 2x)^{\frac{1}{x^2}}=e^2$

៨/របៀបគណនាលីមីតរាងមិនកំនត់ដោយប្រើទ្រឹស្តីបទឡូពីតាល់

ក-ទ្រឹស្តីបទ De l'hospital

សន្មតថាគេមានអនុគមន៍ពីរ $f(x)$ និង $g(x)$ មានដេរីវេ ត្រង់ចំនុច $x = x_0$ ហើយ $g'(x_0) \neq 0$ ។

បើ $\lim\limits_{x \to x_0} \frac{f(x)}{g(x)}$ មានរាង $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ នោះគេបាន

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)} = \frac{f'(x_0)}{g'(x_0)}$$ ដែល $g'(x_0) \neq 0$ ។

ប្រសិនបើ $\lim\limits_{x \to x_0} \frac{f'(x)}{g'(x)}$ មានរាងមិនកំនត់ $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ ដដែល ហើយផលធៀប $\frac{f''(x)}{g''(x)}$ ផ្ទៀងផ្ទាត់លក្ខខណ្ឌឡូពីតាល់ នោះគេអាចអនុវត្តន៍តាមវិធានទូទៅដូចខាងក្រោម

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)} = \lim_{x \to x_0} \frac{f''(x)}{g''(x)} = ... = \lim_{x \to x_0} \frac{f^{(n)}(x)}{g^{(n)}(x)}$$

ខ-រូបមន្តដេរីវេសំខាន់ៗ

អនុគមន៍	ដេរីវេ
1. $y = k$	$y' = 0$
2. $y = x^n$	$y' = n\,x^{n-1}$
3. $y = \frac{1}{x}$	$y' = -\frac{1}{x^2}$

4. $y=\sqrt{x}$ $\qquad$ $y'=\frac{1}{2\sqrt{x}}$

5. $y=e^x$ $\qquad$ $y'=e^x$

6. $y=a^x$ $\qquad$ $y'=a^x\ln a$

7. $y=\ln x$ $\qquad$ $y'=\frac{1}{x}$

8. $y=\sin x$ $\qquad$ $y=\cos x$

9. $y=\cos x$ $\qquad$ $y'=-\sin x$

10. $y=\tan x$ $\qquad$ $y'=\frac{1}{\cos^2 x}=1+\tan^2 x$

11. $y=\cot x$ $\qquad$ $y'=-\frac{1}{\sin^2 x}=-(1+\cot^2 x)$

12. $y=\arcsin x$ $\qquad$ $y'=\frac{1}{\sqrt{1-x^2}}$

13. $y=\arccos x$ $\qquad$ $y'=-\frac{1}{\sqrt{1-x^2}}$

14. $y=\arctan x$ $\qquad$ $y'=\frac{1}{1+x^2}$

គ-រូបមន្តដេរីវេផ្សេងៗទៀត

អនុគមន៍	ដេរីវេ
1. $y=u^n$	$y'=n.u'.u^{n-1}$
2. $y=\sqrt{u}$	$y'=\frac{u'}{2\sqrt{u}}$

3. $y = u.v$ $\quad$ $y' = u'v + v'u$

4. $y = \frac{u}{v}$ $\quad$ $y' = \frac{u'v - v'u}{v^2}$

5. $y = \ln u$ $\quad$ $y' = \frac{u'}{u}$

6. $y = \sin u$ $\quad$ $y' = u'.\cos u$

7. $y = \cos u$ $\quad$ $y' = -u'\sin u$

8. $y = e^u$ $\quad$ $y' = u'.e^u$

9. $y = \tan u$ $\quad$ $y' = u'(1 + \tan^2 u)$

10. $y = \arcsin u$ $\quad$ $y' = \frac{u'}{\sqrt{1-u^2}}$

11. $y = \arccos u$ $\quad$ $y' = -\frac{u'}{\sqrt{1-u^2}}$

12. $y = \arctan u$ $\quad$ $y' = \frac{u'}{1+u^2}$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

១/ $\lim\limits_{x\to 1} \frac{2x^5 + 3x^4 - 5}{x^2 - 1}$ មានរាង $\frac{0}{0}$

$$= \lim_{x\to 1} \frac{(2x^5 + 3x^4 - 5)'}{(x^2 - 1)'} = \lim_{x\to 1} \frac{10x^4 + 12x^3}{2x} = \frac{10+12}{2} = 11$$

ដូចនេះ $\lim\limits_{x\to 1} \frac{2x^5 + 3x^4 - 5}{x^2 - 1} = 11$ ។

២/ $\lim_{x\to 2}\frac{x^4+x^3-x-22}{x^2-4}$ មានរាង $\frac{0}{0}$

$$=\lim_{x\to 2}\frac{(x^4+x^3-x-22)'}{(x^2-4)'}=\lim_{x\to 2}\frac{4x^3+3x^2}{2x}$$

$$=\frac{4(2)^3+3(2)^2}{2(2)}=\frac{32+12}{4}=11$$

ដូចនេះ $\lim_{x\to 2}\frac{x^4+x^3-x-22}{x^2-4}=11$

៣/ $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}$ រាង $\frac{0}{0}$

$$=\lim_{x\to 1}\frac{(x^3-1)'}{(\sqrt{x}-1)'}=\lim_{x\to 1}\frac{3x^2}{\frac{1}{2\sqrt{x}}}=\frac{3}{\frac{1}{2}}=6$$

ដូចនេះ $\lim_{x\to 1}\frac{x^3-1}{\sqrt{x}-1}=6$

៤/ $\lim_{x\to 1}\frac{\sin(\pi x)}{1-x^3}$ រាង $\frac{0}{0}$

$$=\lim_{x\to 1}\frac{(\sin\pi x)'}{(1-x^3)'}=\lim_{x\to 1}\frac{\pi\cos\pi x}{-3x^2}=\frac{\pi\cos\pi}{-3}=\frac{\pi}{3}$$

ដូចនេះ $\lim_{x\to 1}\frac{\sin(\pi x)}{1-x^3}=\frac{\pi}{3}$

៥/ $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$ រាង $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{(x-\sin x)'}{(x^3)'}=\lim_{x\to 0}\frac{1-\cos x}{3x^2}$$

$$=\lim_{x\to 0}\frac{(1-\cos x)'}{(3x^2)'}=\lim_{x\to 0}\frac{\sin x}{6x}=\frac{1}{6}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}=\dfrac{1}{6}$

៦/ $\lim\limits_{x\to 0}\dfrac{\ln(\cos x)}{x^2}$ រាង $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{(\ln\cos x)'}{(x^2)'}=\lim_{x\to 0}\frac{-\dfrac{\sin x}{\cos x}}{2x}$$

$$=-\frac{1}{2}\lim_{x\to 0}\frac{\tan x}{x}=-\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{\ln(\cos x)}{x^2}=-\dfrac{1}{2}$

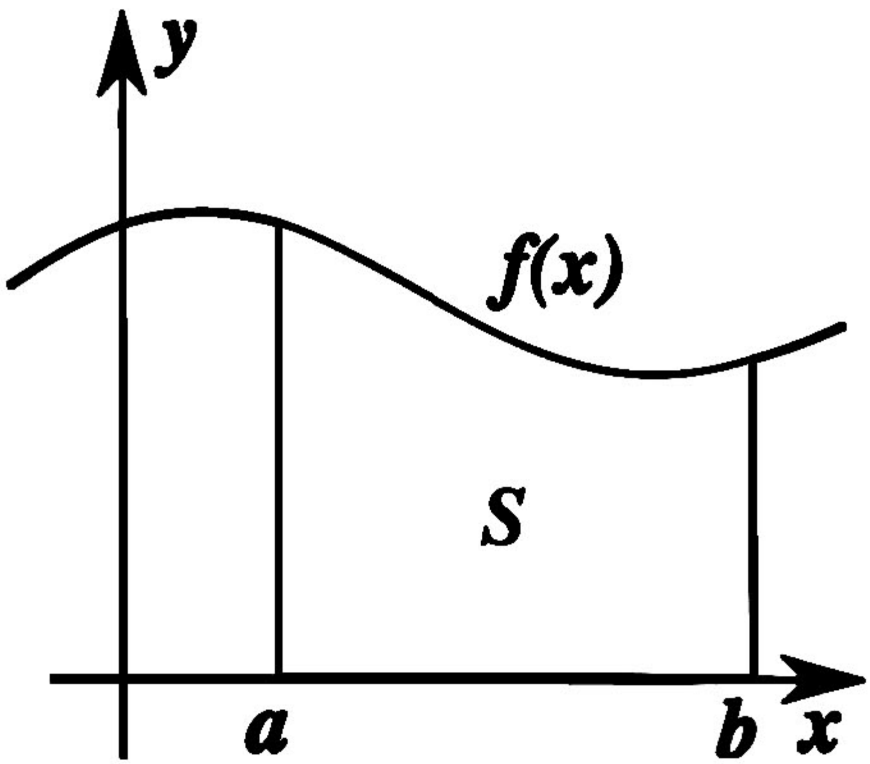
៨/ $\lim\limits_{x\to 0}\dfrac{e^x+\ln(1+x)-1}{1+x-\cos x}$ រាង $\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{e^x+\dfrac{1}{1+x}}{1+\sin x}=\frac{1+1}{1+0}=2$$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{e^x+\ln(1+x)-1}{1+x-\cos x}=2$

៨/របៀបគណនាលីមីតដោយប្រើនិយមន័យអាំងតេក្រាលកំនត់

ក-និយមន័យ



គេមានអនុគមន៍ f(x) ដែលជាប់នៅចន្លោះ [a, b],

គេចែកចន្លោះ[a, b] ជា n ផ្នែកស្មើៗគ្នាតាមលំដាប់

$x_0 = a$, $x_1, x_2, ..., x_n = b$ ដោយចំនុច $x_k = a + \frac{b-a}{n}k$

ដែល $\Delta x_k = x_k - x_{k-1} = \frac{b-a}{n}$ នោះគេបាន

$$\int_a^b f(x).dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(x_k)\Delta x_k = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x_k$$

ខ- រូបមន្ត Newton-Leibnitz

គេអោយអនុគមន៍ f(x) ជាអនុគមន៍កំនត់និងជាប់

លើ [a, b] និង F(x) ជាព្រីមីទីវនៃអនុគមន៍ f(x)។

គេបាន $\int_a^b f(x).dx = [F(x)]_a^b = F(b) - F(a)$

គ-លក្ខណៈនៃអាំងតេក្រាលកំនត់

១. $\int_a^a f(x).dx = 0$

២. $\int_a^b f(x).dx = -\int_b^a f(x).dx$

៣. $\int_a^b [\, f(x) + g(x) \,] .dx = \int_a^b f(x).dx + \int_a^b g(x).dx$

៤. $\int_a^b k\, f(x).dx = k \int_a^b f(x).dx$

៥. $\int_a^c f(x).dx + \int_c^b f(x).dx = \int_a^b f(x).dx \quad ; \; (\; a < c < b \;)$

ឃ-អាំងតេក្រាលដោយផ្នែក $\int_a^b U\, dV = [\; UV \;]_a^b - \int_a^b V\, dU$

ឧទាហរណ៍ គណនាលីមីតខាងក្រោម

ក/ $\lim\limits_{n\to+\infty}\frac{1^5+2^5+3^5+.....+n^5}{n^6}$

តាង $S_n=\frac{1^5+2^5+3^5+....+n^5}{n^6}$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^5=\sum_{k=1}^{n}\left[\left(\frac{k}{n}\right)^5.\frac{1}{n}\right]$$

យក $x_k=\frac{k}{n}$ នោះ $\Delta x_k=x_k-x_{k-1}=\frac{k}{n}-\frac{k-1}{n}=\frac{1}{n}$

និងអនុគមន៍ $f(x)=x^5$ នោះ $f(x_k)=\left(\frac{k}{n}\right)^5$

គេបាន $S_n=\sum_{k=1}^{n}[f(x_k)\Delta x_k]$

តាមនិយមន័យអាំងតេក្រាលកំនត់គេបាន

$$\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\sum_{k=1}^{n}[f(x_k)\Delta x_k]=\int_0^1 f(x).dx$$

$$=\int_0^1 x^5dx=\left[\frac{1}{6}x^6\right]_0^1=\frac{1}{6}$$

ដូចនេះ $\lim\limits_{n\to+\infty}\frac{1^5+2^5+3^5+.....+n^5}{n^6}=\frac{1}{6}$

ខ/ $\lim_{n\to+\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{n+n}\right)$

តាង $S_n=\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{n+n}$

$$=\frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+.....+\frac{1}{1+\frac{n}{n}}\right)$$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\frac{1}{1+\frac{k}{n}}\right)=\sum_{k=1}^{n}\left[\frac{1}{1+\frac{k}{n}}.\frac{1}{n}\right]$$

យក $x_k=\frac{k}{n}$ នោះ $\Delta x_k=x_k-x_{k-1}=\frac{k}{n}-\frac{k-1}{n}=\frac{1}{n}$

និងអនុគមន៍ $f(x)=\frac{1}{1+x}$ នោះ $f(x_k)=\frac{1}{1+\frac{k}{n}}$

គេបាន $S_n=\sum_{k=1}^{n}[f(x_k)\Delta x_k]$

តាមនិយមន័យអាំងតេក្រាលកំនត់គេបាន

$$\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\sum_{k=1}^{n}[f(x_k)\Delta x_k]=\int_0^1 f(x).dx$$

$$=\int_0^1\frac{1}{1+x}dx=[\ln|1+x|]_0^1=\ln 2$$

ដូចនេះ $\lim_{n\to+\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{n+n}\right)=\ln 2$

គ/ $\lim_{n\to+\infty}\frac{1}{n}\left(\tan\frac{a}{n}+\tan\frac{2a}{n}+.....+\tan\frac{na}{n}\right)$

តាង $S_n=\frac{1}{n}\left(\tan\frac{a}{n}+\tan\frac{2a}{n}+....+\tan\frac{na}{n}\right)$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\tan\frac{ka}{n}\right)=\sum_{k=1}^{n}\left[\tan\frac{ka}{n}.\frac{1}{n}\right]$$

យក $x_k=\frac{k}{n}$ នោះ $\Delta x_k=x_k-x_{k-1}=\frac{k}{n}-\frac{k-1}{n}=\frac{1}{n}$

និងអនុគមន៍ $f(x)=\tan(ax)$ នោះ $f(x_k)=\tan\frac{ka}{n}$

គេបាន $S_n=\sum_{k=1}^{n}[f(x_k)\Delta x_k]$

តាមនិយមន័យអាំងតេក្រាលកំនត់គេបាន

$$\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\sum_{k=1}^{n}[f(x_k)\Delta x_k]=\int_0^1 f(x).dx$$

$$=\int_0^1\tan(ax)dx=\left[-\frac{\ln(\cos ax)}{a}\right]_0^1=-\frac{\ln(\cos a)}{a}$$

ដូចនេះ $\lim_{n\to+\infty}\frac{1}{n}\left(\tan\frac{a}{n}+\tan\frac{2a}{n}+.....+\tan\frac{na}{n}\right)=-\frac{\ln(\cos a)}{a}$

ឃ/ $\lim_{n\to+\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+4}+......+\frac{n}{n^2+n^2}\right)$

តាង $S_n=\frac{n}{n^2+1}+\frac{n}{n^2+4}+.....+\frac{n}{n^2+n^2}$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\frac{1}{1+(\frac{k}{n})^2}\right)=\sum_{k=1}^{n}\left[\frac{1}{1+(\frac{k}{n})^2}.\frac{1}{n}\right]$$

យក $x_k=\frac{k}{n}$ នោះ $\Delta x_k=x_k-x_{k-1}=\frac{k}{n}-\frac{k-1}{n}=\frac{1}{n}$

និងអនុគមន៍ $f(x)=\frac{1}{1+x^2}$ នោះ $f(x_k)=\frac{1}{1+(\frac{k}{n})^2}$

គេបាន $S_n=\sum_{k=1}^{n}[f(x_k)\Delta x_k]$

តាមនិយមន័យអាំងតេក្រាលកំនត់គេបាន

$$\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\sum_{k=1}^{n}[f(x_k)\Delta x_k]=\int_0^1 f(x).dx$$

$$=\int_0^1\frac{1}{1+x^2}dx=[\arctan x]_0^1$$

$$=\arctan(1)-\arctan(0)=\frac{\pi}{4}$$

ដូចនេះ $\lim_{n\to+\infty}\left(\frac{n}{n^2+1}+\frac{n}{n^2+4}+......+\frac{n}{n^2+n^2}\right)=\frac{\pi}{4}$

ច/ $\lim_{n\to+\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+4}}+......+\frac{1}{\sqrt{n^2+n^2}}\right)$

តាង $S_n=\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+4}}+.....+\frac{1}{\sqrt{n^2+n^2}}$

$$=\frac{1}{n}\sum_{k=1}^{n}\left(\frac{1}{\sqrt{1+(\frac{k}{n})^2}}\right)=\sum_{k=1}^{n}\left[\frac{1}{\sqrt{1+(\frac{k}{n})^2}}.\frac{1}{n}\right]$$

យក $x_k=\frac{k}{n}$ នោះ $\Delta x_k=x_k-x_{k-1}=\frac{k}{n}-\frac{k-1}{n}=\frac{1}{n}$

និងអនុគមន៍ $f(x)=\frac{1}{\sqrt{1+x^2}}$ នោះ $f(x_k)=\frac{1}{\sqrt{1+(\frac{k}{n})^2}}$

គេបាន $S_n=\sum_{k=1}^{n}\left[f(x_k)\Delta x_k\right]$

តាមនិយមន័យអាំងតេក្រាលកំនត់គេបាន

$$\lim_{n\to+\infty}S_n=\lim_{n\to+\infty}\sum_{k=1}^{n}\left[f(x_k)\Delta x_k\right]=\int_0^1 f(x).dx$$

$$=\int_0^1\frac{1}{\sqrt{1+x^2}}dx=\left[\ln(x+\sqrt{1+x^2})\right]_0^1$$

$$=\ln(1+\sqrt{2})$$

ដូចនេះ

$$\lim_{n\to+\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+4}}+......+\frac{1}{\sqrt{n^2+n^2}}\right)=1+\sqrt{2}$$

ជំពូកទី៣

លំហាត់សារឡើងវិញ

លំហាត់ទី១

ចូរគណនាលីមីតខាងក្រោម

ក/ $\lim_{x\to 1}\frac{x^3-2x^2+x}{\sqrt{x^2+3}-\sqrt{2x+2}}$

ខ/ $\lim_{x\to 2}\frac{x-2}{\sqrt{x^3+8}-4}$

គ/ $\lim_{x\to 1}\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}$

ឃ/ $\lim_{x\to 3}\frac{x^2-9}{x-1-\sqrt{x+1}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម

ក/ $\lim_{x\to 1}\frac{x^3-2x^2+x}{\sqrt{x^2+3}-\sqrt{2x+2}}$

$$= \lim_{x\to1}\frac{x(x-1)^2(\sqrt{x^2+3}+\sqrt{2x+2})}{(x^2+3)-(2x+2)}$$

$$= \lim_{x\to1}\frac{x(x-1)^2(\sqrt{x^2+3}+\sqrt{2x+2})}{(x-1)^2}$$

$$= \lim_{x\to1}\left[x(\sqrt{x^2+3}+\sqrt{2x+2})\right] = 2+2=4$$

ដូចនេះ $\lim_{x\to1}\frac{x^3-2x^2+x}{\sqrt{x^2+3}-\sqrt{2x+2}}=4$

ខ/ $\lim_{x\to2}\frac{x-2}{\sqrt{x^3+8}-4}$

$$= \lim_{x\to2}\frac{(x-2)(\sqrt{x^3+8}+4)}{(x^3+8)-16}$$

$$= \lim_{x\to2}\frac{(x-2)(\sqrt{x^3+8}+4)}{(x-2)(x^2+2x+4)}$$

$$= \lim_{x\to2}\frac{\sqrt{x^3+8}+4}{x^2+2x+4} = \frac{4+4}{4+4+4} = \frac{2}{3}$$

ដូចនេះ $\lim_{x\to2}\frac{x-2}{\sqrt{x^3+8}-4}=\frac{2}{3}$

គ/ $\lim_{x\to1}\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}$

$$= \lim_{x\to1}\frac{\left[(x+1)^2-(x^2+3)\right]\left(\sqrt{x}+1\right)}{(x-1)(x+1+\sqrt{x^2+3})}$$

$$= \lim_{x\to 1} \frac{2(x-1)(\sqrt{x}+1)}{(x-1)(x+1+\sqrt{x^2+3})}$$

$$= \lim_{x\to 1} \frac{2(\sqrt{x}+1)}{x+1+\sqrt{x^2+3}} = \frac{4}{2+2} = 1$$

ដូចនេះ $\lim_{x\to 1} \frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1} = 1$

ឃ/ $\lim_{x\to 3} \frac{x^2-9}{x-1-\sqrt{x+1}}$

$$= \lim_{x\to 3} \frac{(x^2-9)(x-1+\sqrt{x+1})}{(x-1)^2-(x+1)}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+3)(x-1+\sqrt{x+1})}{x(x-3)}$$

$$= \lim_{x\to 3} \frac{(x+3)(x-1+\sqrt{x+1})}{x} = \frac{6\times 4}{3} = 8$$

ដូចនេះ $\lim_{x\to 3} \frac{x^2-9}{x-1-\sqrt{x+1}} = 8$ ។

លំហាត់ទី២

គេឱ្យអនុគមន៍ $f(x)=\dfrac{x^5+ax^2+bx+2}{x-1}$

ក. ចូរគណនា $\lim\limits_{x\to 1} f(x)$ ចំពោះ $a=-1$, $b=-2$ ។

ខ. កំនត់តម្លៃ a និង b ដើម្បីឱ្យ $\lim\limits_{x\to 1} f(x)=10$ ។

ដំណោះស្រាយ

ក. គណនា $\lim\limits_{x\to 1} f(x)$:

ចំពោះ $a=-1$, $b=-2$ យើងបាន :

$$\lim_{x\to 1} f(x)=\lim_{x\to 1}\frac{x^5-x^2-2x+2}{x-1}$$

$$=\lim_{x\to 1}\frac{x^2(x-1)(x^2+x+1)-2(x-1)}{x-1}$$

$$=\lim_{x\to 1}\left[x^2(x^2+x+1)-2\right]=3-2=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1} f(x)=\lim\limits_{x\to 1}\dfrac{x^5-x^2-2x+2}{x-1}=1}$ ។

ខ. កំនត់តម្លៃ a និង b

យើងមាន $f(x)=\dfrac{x^5+ax^2+bx+2}{x-1}$

$$= \frac{(x^5 - 1) + a(x^2 - 1) + b(x - 1) + (a + b + 3)}{x - 1}$$

$$= x^4 + x^3 + x^2 + x + 1 + a(x + 1) + b + \frac{a + b + 3}{x - 1}$$

ដើម្បីឱ្យ $\lim_{x \to 1} f(x) = 10$ លុះត្រាតែ $a + b + 3 = 0$ ឬ $a + b = -3$ (1)

និង $\lim_{x \to 1} \left[x^4 + x^3 + x^2 + x + 1 + a(x + 1) + b\right] = 10$

ឬ $5 + 2a + b = 10$ នាំឱ្យ $2a + b = 5$ (2) ។

ដកសមីការ (1) និង (2) គេបាន $-a = -8$ នាំឱ្យ $a = 8$

និង $b = -3 - a = -11$ ។

ដូចនេះ $a = 8$; $b = -11$ ។

លំហាត់ទី៣

គណនាលីមីត

ក. $\lim\limits_{x\to 2}(x^2-x-2)\tan\frac{\pi}{x}$

ខ. $\lim\limits_{x\to 1}\frac{1-x^3}{\cos\frac{\pi}{x+1}}$

គ. $\lim\limits_{x\to 2}\frac{(2-x)^2}{1-\sin\frac{\pi}{x}}$

ឃ. $\lim\limits_{x\to \pi}\frac{\cos\frac{\pi x}{x+\pi}}{\pi-x}$

ង. $\lim\limits_{x\to 3}\frac{x-3}{\cos\frac{\pi x}{x+3}}$

ច. $\lim\limits_{x\to 1}\frac{\tan\frac{\pi}{x}}{1-x^2}$

ឆ. $\lim\limits_{x\to 1}\frac{1-\sin\frac{2\pi}{x+3}}{(1-x)^2}$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to 2}(x^2-x-2)\tan\frac{\pi}{x}$

តាង $z=\frac{1}{x}$ នាំឱ្យ $x=\frac{1}{z}$ កាលណា, $x\to 2$ នោះ $z\to\frac{1}{2}$

$$=\lim_{z\to\frac{1}{2}}\left(\frac{1}{z^2}-\frac{1}{z}-2\right)\tan\pi z$$

តាង $u=\frac{1}{2}-z$ នាំឱ្យ $z=\frac{1}{2}-u$ កាលណា

$z\to\frac{1}{2}$ នោះ $u\to 0$

$$=\lim_{u\to 0}\left[\frac{1}{(\frac{1}{2}-u)^2}-\frac{1}{\frac{1}{2}-u}-2\right]\tan\pi(\frac{1}{2}-u)$$

$$=\lim_{u\to 0}\frac{1-\frac{1}{2}+u-2(\frac{1}{2}-u)^2}{(\frac{1}{2}-u)^2}\tan(\frac{\pi}{2}-\pi u)$$

$$=\lim_{u\to 0}\frac{3u-2u^2}{(\frac{1}{2}-u)^2}\cot(\pi u)$$

$$=\lim_{u\to 0}\frac{3-2u}{(0.5-u)^2}.\frac{\pi u}{\tan\pi u}.\frac{1}{\pi}=\frac{12}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to 2}(x^2-x-2)\tan\frac{\pi}{x}=\frac{12}{\pi}}$ ។

ខ. $\lim_{x\to 1}\frac{1-x^3}{\cos\frac{\pi}{x+1}}$

តាង $z=\frac{1}{x+1}$ នាំឱ្យ $x=\frac{1}{z}-1$ កាលណា, $x\to 1$ នោះ $z\to\frac{1}{2}$

$$=\lim_{z\to\frac{1}{2}}\frac{1-(\frac{1}{z}-1)^3}{\cos\pi z}$$

តាង $u=\frac{1}{2}-z$ នាំឱ្យ $z=\frac{1}{2}-u$ កាលណា

$z\to\frac{1}{2}$ នោះ $u\to 0$

$$=\lim_{u\to 0}\frac{1-(\frac{1}{\frac{1}{2}-u}-1)^3}{\cos(\frac{\pi}{2}-\pi u)}$$

$$=\lim_{u\to 0}\frac{(\frac{1}{2}-u)^3-(1-\frac{1}{2}+u)^3}{(\frac{1}{2}-u)^3\sin\pi u}$$

$$=\lim_{u\to 0}\frac{\frac{1}{8}-\frac{3}{4}u+\frac{3}{2}u^2-u^3-\frac{1}{8}-\frac{3}{4}u-\frac{3}{2}u^2-u^3}{(\frac{1}{2}-u)^3\sin\pi u}$$

$$=\lim_{u\to 0}\frac{-\frac{3}{2}u-2u^3}{(\frac{1}{2}-u)^3\sin\pi u}=\lim_{u\to 0}\frac{-\frac{3}{2}-2u^2}{(\frac{1}{2}-u)^3}.\frac{\pi u}{\sin\pi u}.\frac{1}{\pi}=-\frac{12}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{1-x^3}{\cos\frac{\pi}{x+1}}=-\frac{12}{\pi}}$ ។

គ. $\lim_{x\to 2}\frac{(2-x)^2}{1-\sin\frac{\pi}{x}}$

តាង $z=\frac{1}{x}$ នាំឱ្យ $x=\frac{1}{z}$ កាលណា, $x\to 2$ នោះ $z\to\frac{1}{2}$

$$=\lim_{z\to\frac{1}{2}}\frac{(2-\frac{1}{z})^2}{1-\sin\pi z}$$

តាង $u=\frac{1}{2}-z$ នាំឱ្យ $z=\frac{1}{2}-u$

កាលណា $z\to\frac{1}{2}$ នោះ $u\to 0$

$$=\lim_{u\to 0}\frac{(2-\frac{1}{0.5-u})^2}{1-\sin(\frac{\pi}{2}-\pi u)}$$

$$=\lim_{u\to 0}\frac{(1-2u-1)^2}{(0.5-u)^2(1-\cos\pi u)}$$

$$=\lim_{u\to 0}\frac{4u^2}{(0.5-u)^2 2\sin^2\frac{\pi u}{2}}$$

$$=2\lim_{u\to 0}\frac{1}{(0.5-u)^2}\cdot\frac{(\frac{\pi u}{2})^2}{\sin^2\frac{\pi u}{2}}\cdot\frac{4}{\pi^2}=\frac{8}{\pi^2}$$

ដូចនេះ $\boxed{\lim_{x\to 2}\frac{(2-x)^2}{1-\sin\frac{\pi}{x}}=\frac{8}{\pi^2}}$ ។

ឃ. $\lim\limits_{x\to\pi}\dfrac{\cos\dfrac{\pi x}{x+\pi}}{\pi-x}$

តាង $t=\dfrac{\pi x}{x+\pi}\Rightarrow x=\dfrac{\pi t}{\pi-t}$ បើ $x\to\pi\ \Rightarrow t\to\dfrac{\pi}{2}$

$$=\lim_{t\to\frac{\pi}{2}}\frac{\cos t}{\pi-\dfrac{\pi t}{\pi-t}}=\lim_{t\to\frac{\pi}{2}}\frac{(\pi-t)\cos t}{\pi(\pi-2t)}$$

តាង $u=\dfrac{\pi}{2}-t\ \Rightarrow t=\dfrac{\pi}{2}-u$

បើ $t\to\dfrac{\pi}{2}\Rightarrow u\to 0$

$$=\lim_{u\to 0}\frac{(\pi-\dfrac{\pi}{2}+u)\cos(\dfrac{\pi}{2}-u)}{\pi(\pi-\pi+2u)}$$

$$=\lim_{u\to 0}\frac{\dfrac{\pi}{2}+u}{2\pi}.\frac{\sin u}{u}=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\pi}\dfrac{\cos\dfrac{\pi x}{x+\pi}}{\pi-x}=\dfrac{1}{4}}$ ។

ង. $\lim\limits_{x\to 3}\dfrac{x-3}{\cos\dfrac{\pi x}{x+3}}$

តាង $z=\dfrac{\pi x}{x+3}$ នាំឱ្យ $x=\dfrac{3z}{\pi-z}$ កាលណា, $x\to 3$ នោះ $z\to\dfrac{\pi}{2}$

$$=\lim_{z\to\frac{\pi}{2}}\frac{\dfrac{3z}{\pi-z}-3}{\cos z}=\lim_{z\to\frac{\pi}{2}}\frac{3(2z-\pi)}{(\pi-z)\cos z}$$

តាង $u=\frac{\pi}{2}-z \Rightarrow z=\frac{\pi}{2}-u$

បើ $z\to\frac{\pi}{2}\Rightarrow u\to 0$

$$=\lim_{u\to 0}\frac{3(\pi-2u-\pi)}{(\pi-\frac{\pi}{2}+u)\cos(\frac{\pi}{2}-u)}=\lim_{u\to 0}\frac{-6u}{(\frac{\pi}{2}+u)\sin u}$$

$$=\lim_{u\to 0}\frac{-6}{\frac{\pi}{2}+u}.\frac{u}{\sin u}=-\frac{12}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to 3}\frac{x-3}{\cos\frac{\pi x}{x+3}}=-\frac{12}{\pi}}$ ។

ច. $\lim_{x\to 1}\frac{\tan\frac{\pi}{x}}{1-x^2}$

តាង $z=\frac{1}{x}$ នាំឱ្យ $x=\frac{1}{z}$

កាលណា $x\to 1$ នោះ $z\to 1$

$$=\lim_{z\to 1}\frac{\tan\pi z}{1-\frac{1}{z^2}}=\lim_{z\to 1}\frac{z^2\tan\pi z}{z^2-1}$$

តាង $u=1-z$ នាំឱ្យ $z=1-u$

កាលណា $z\to 1$ នោះ $u\to 0$

$$= \lim_{u\to 0} \frac{(1-u)^2 \tan(\pi - \pi u)}{(1-u)^2 - 1}$$

$$= \lim_{u\to 0} \frac{(1-u)^2(-\tan \pi u)}{2u - u^2}$$

$$= \lim_{u\to 0} \frac{-(1-u)^2}{2-u} . \frac{\tan \pi u}{\pi u} . \pi = -\frac{\pi}{2}$$

ដូចនេះ $\boxed{\lim_{x\to 1} \frac{\tan \frac{\pi}{x}}{1-x^2} = -\frac{\pi}{2}}$ ។

ឆ. $\lim_{x\to 1} \frac{1 - \sin \frac{2\pi}{x+3}}{(1-x)^2}$

តាង $z = \frac{1}{x+3}$ នាំឱ្យ $x = \frac{1}{z} - 3$

កាលណា, $x \to 1$ នោះ $z \to \frac{1}{4}$

$$= \lim_{z\to \frac{1}{4}} \frac{1 - \sin 2\pi z}{(1 - \frac{1}{z} + 3)^2} = \lim_{z\to \frac{1}{4}} \frac{z^2(1 - \sin 2\pi z)}{(4z-1)^2}$$

តាង $u = \frac{1}{4} - z$ នាំឱ្យ $z = \frac{1}{4} - u$

កាលណា, $z \to \frac{1}{4}$ នោះ $u \to 0$

$$= \lim_{u\to 0} \frac{(0.25-u)^2[1-\sin(\frac{\pi}{2}-\pi u)]}{(1-4u-1)^2}$$

$$= \lim_{u\to 0} \frac{(0.25-u)^2(1-\cos\pi u)}{16u^2}$$

$$= \lim_{u\to 0} \frac{(0.25-u)^2.2\sin^2\frac{\pi u}{2}}{16u^2}$$

$$= \frac{1}{8}\lim_{u\to 0}(0.25-u)^2.\frac{\sin^2\frac{\pi u}{2}}{(\frac{\pi u}{2})^2}.\frac{\pi^2}{4} = \frac{1}{8}.\frac{1}{16}.\frac{\pi^2}{4} = \frac{\pi^2}{512}$$

ដូចនេះ $\boxed{\lim_{x\to 1} \frac{1-\sin\frac{2\pi}{x+3}}{(1-x)^2} = \frac{\pi^2}{512}}$ ។

<u>លំហាត់ទី៤</u>

គណនាលីមីតខាងក្រោម :

ក. $\lim_{x\to 1} \frac{1-\sin\frac{\pi x}{2}}{(1-x)^2}$

ខ. $\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\pi^2-4x^2}$

គ. $\lim_{x\to 2}(4-x^2)\tan\frac{\pi x}{4}$

ឃ. $\lim_{x\to \pi}(\pi-x)\tan\frac{x}{2}$

គ. $\lim_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{\pi-4x}$

ង. $\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{1-\sqrt{2}\sin x}$

ឃ. $\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$

ឈ. $\lim_{x\to\frac{\pi}{3}} \frac{\pi-3x}{\sqrt{3}-2\sin x}$

ង. $\lim_{x\to 1} \frac{x^3-1+\tan \pi x}{1-x^2}$

ញ. $\lim_{x\to 2} \frac{x^3-8}{\cos\frac{\pi}{x}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម :

ក. $\lim_{x\to 1} \frac{1-\sin\frac{\pi x}{2}}{(1-x)^2}$

តាង $x=1-z$ កាលណា $x\to 1$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{1-\sin(\frac{\pi}{2}-\frac{\pi z}{2})}{z^2}=\lim_{z\to 0}\frac{1-\cos\frac{\pi z}{2}}{z^2}$$

$$=\lim_{z\to 0}\frac{2\sin^2\frac{\pi z}{4}}{z^2}=2\lim_{z\to 0}\frac{\sin^2\frac{\pi z}{4}}{(\frac{\pi z}{4})^2}.\frac{\pi^2}{16}=\frac{\pi^2}{8}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{1-\sin\frac{\pi x}{2}}{(1-x)^2}=\frac{\pi^2}{8}}$ ។

ខ. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\pi^2 - 4x^2}$

តាង $z = \frac{\pi}{2} - x$ នាំឱ្យ $x = \frac{\pi}{2} - z$ កាលណា $x \to \frac{\pi}{2}$ នោះ $z \to 0$

$$= \lim_{z\to 0} \frac{\cos(\frac{\pi}{2} - z)}{\pi^2 - 4(\frac{\pi}{2} - z)^2}$$

$$= \lim_{z\to 0} \frac{\sin z}{\pi^2 - \pi^2 + 4\pi z - 4z^2}$$

$$= \lim_{z\to 0} \frac{\sin z}{4\pi z - 4z^2}$$

$$= \lim_{z\to 0} \frac{\sin z}{z} . \frac{1}{4\pi - 4z} = \frac{1}{4\pi}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\pi^2 - 4x^2} = \frac{1}{4\pi}}$ ។

គ. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{\pi - 4x}$

តាង $z = \frac{\pi}{4} - x$ នាំឱ្យ $x = \frac{\pi}{4} - z$

កាលណា $x \to \frac{\pi}{4}$ នោះ $z \to 0$

$$= \lim_{z\to 0}\frac{\sin(\frac{\pi}{4}-z)-\cos(\frac{\pi}{4}-z)}{\pi-4(\frac{\pi}{4}-z)}$$

$$= \lim_{z\to 0}\frac{(\frac{\sqrt{2}}{2}\cos z-\frac{\sqrt{2}}{2}\sin z)-(\frac{\sqrt{2}}{2}\cos z+\frac{\sqrt{2}}{2}\sin z)}{4z}$$

$$= \lim_{z\to 0}\frac{-\sqrt{2}\sin z}{4z} = -\frac{\sqrt{2}}{4}\lim_{z\to 0}\frac{\sin z}{z} = -\frac{\sqrt{2}}{4}$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\pi-4x} = -\frac{\sqrt{2}}{4}}$

ឃ. $\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$

តាង $z=\frac{\pi}{2}-x$ នាំឱ្យ $x=\frac{\pi}{2}-z$

កាលណា $x\to\frac{\pi}{2}$ នោះ $z\to 0$

$$= \lim_{z\to 0}\frac{\sqrt{2}-\sqrt{1+\sin(\frac{\pi}{2}-z)}}{\cos^2(\frac{\pi}{2}-z)}$$

$$= \lim_{z\to 0}\frac{\sqrt{2}-\sqrt{1+\cos z}}{\sin^2 z}$$

$$= \lim_{z\to 0} \frac{2-1-\cos z}{\sin^2 z.(\sqrt{2}+\sqrt{1+\cos z})}$$

$$= \lim_{z\to 0} \frac{1-\cos z}{\sin^2 z(\sqrt{2}+\sqrt{1+\cos z})}$$

$$= \lim_{z\to 0} \frac{2\sin^2 \frac{z}{2}}{\sin^2 z.(\sqrt{2}+\sqrt{1+\cos z})}$$

$$= 2\lim_{z\to 0} \frac{\sin^2 \frac{z}{2}}{(\frac{z}{2})^2}.\frac{z^2}{\sin^2 z}.\frac{1}{4}.\frac{1}{\sqrt{2}+\sqrt{1+\cos z}}$$

$$= 2.\frac{1}{4}.\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{8}$$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} = \frac{\sqrt{2}}{8}}$ ។

ង. $\lim_{x\to 1} \frac{x^3-1+\tan \pi x}{1-x^2}$

តាង $x = 1-z$

កាលណា $x \to 1$ នោះ $z \to 0$

$$= \lim_{z\to 0}\frac{(1-z)^3-1+\tan(\pi-\pi z)}{1-(1-z)^2}$$

$$= \lim_{z\to 0}\frac{1-3z+3z^2-z^3-1-\tan\pi z}{1-1+2z-z^2}$$

$$= \lim_{z\to 0}\frac{-3z+3z^2-z^3-\tan\pi z}{2z-z^2}$$

$$= \lim_{z\to 0}\frac{z(-3+3z-z^2-\frac{\tan\pi z}{z})}{z(2-z)} = \frac{-3-\pi}{2}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x^3-1+\tan\pi x}{1-x^2} = -\frac{3+\pi}{2}}$

ច. $\lim_{x\to 2}(4-x^2)\tan\frac{\pi x}{4}$

តាង $z=2-x$ នាំឱ្យ $x=2-z$

កាលណា $x\to 2$ នោះ $z\to 0$

$$= \lim_{z\to 0}\left[4-(2-z)^2\right]\tan\frac{\pi}{4}(2-z)$$

$$= \lim_{z\to 0}(4z-z^2)\tan(\frac{\pi}{2}-\frac{\pi z}{4})$$

$$= \lim_{z\to 0}(4-z)\frac{\frac{\pi z}{4}}{\tan\frac{\pi z}{4}}.\frac{4}{\pi} = \frac{16}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to 2}(4-x^2)\tan\frac{\pi x}{4} = \frac{16}{\pi}}$ ។

ឆ. $\lim\limits_{x\to\pi}(\pi-x)\tan\frac{x}{2}$

តាង $z=\pi-x$ នាំឱ្យ $x=\pi-z$

កាលណា $x\to\pi$ នោះ $z\to 0$

$$=\lim_{z\to 0} z\tan\frac{\pi-z}{2}$$

$$=\lim_{z\to 0} z\tan(\frac{\pi}{2}-\frac{z}{2})$$

$$=\lim_{z\to 0} z\cot z=\lim_{z\to 0}\frac{z}{\tan z}=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to\pi}(\pi-x)\tan\frac{x}{2}=1}$ ។

ជ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}$

តាង $z=\frac{\pi}{4}-x$ នាំឱ្យ $x=\frac{\pi}{4}-z$ កាលណា, $x\to\frac{\pi}{4}$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{1-\tan(\frac{\pi}{4}-z)}{1-\sqrt{2}\sin(\frac{\pi}{4}-z)}=\lim_{z\to 0}\frac{1-\frac{1-\tan z}{1+\tan z}}{1-\sqrt{2}(\frac{\sqrt{2}}{2}\cos z-\frac{\sqrt{2}}{2}\sin z)}$$

$$=\lim_{z\to 0}\frac{2\tan z}{(1-\cos z+\sin z)(1+\tan z)}$$

$$=2\lim_{z\to 0}\frac{\frac{\tan z}{z}}{(\frac{1-\cos z}{z}+\frac{\sin z}{z})(1+\tan z)}=2.\frac{1}{(0+1)(1+0)}=2$$

ដូចនេះ $$\boxed{\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{1-\sqrt{2}\sin x}=2}$$ ។

ឈ. $\lim_{x\to\frac{\pi}{3}}\frac{\pi-3x}{\sqrt{3}-2\sin x}$

តាង $z=\frac{\pi}{3}-x$ នាំឱ្យ $x=\frac{\pi}{3}-z$ កាលណា, $x\to\frac{\pi}{3}$ នោះ $z\to 0$

$$=\lim_{z\to 0}\frac{\pi-3(\frac{\pi}{3}-z)}{\sqrt{3}-2\sin(\frac{\pi}{3}-z)}$$

$$=\lim_{z\to 0}\frac{3z}{\sqrt{3}-2(\frac{\sqrt{3}}{2}\cos z-\frac{1}{2}\sin z)}$$

$$=\lim_{z\to 0}\frac{3z}{\sqrt{3}-\sqrt{3}\cos z-\sin z}$$

$$=\lim_{z\to 0}\frac{3}{\sqrt{3}\frac{1-\cos z}{z}-\frac{\sin z}{z}}=\frac{3}{0-1}=-3$$

ដូចនេះ $$\boxed{\lim_{x\to\frac{\pi}{3}}\frac{\pi-3x}{\sqrt{3}-2\sin x}=-3}$$ ។

លំហាត់ទី៥

គេឱ្យអនុគមន៍ $f(x)=\dfrac{x^3+px^2+q}{x^2-2x+2}$

ក. ចូរបង្ហាញថាអនុគមន៍ f កំណត់បានជានិច្ចចំពោះគ្រប់ $x\in IR$ ។

ខ. កំណត់ចំនួនពិត p និង q បើគេដឹងថាខ្សែកោង (c) តាងអនុគមន៍ f

មានបន្ទាត់ $y=x+2$ ជាអាស៊ីមតូតទ្រេត ហើយកាត់តាមចំនុច $A(2,4)$ ។

ដំណោះស្រាយ

ក. បង្ហាញថាអនុគមន៍ f កំណត់បានជានិច្ច :

គេមាន $f(x)=\dfrac{x^3+px+q}{x^2-2x+2}$

ដោយ $x^2-2x+2=(x^2-2x+1)+1=(x-1)^2+1>0,\forall x\in IR$

ដូចនេះ អនុគមន៍f កំណត់បានជានិច្ចចំពោះគ្រប់ $x\in IR$ ។

ខ. កំណត់ចំនួនពិត p និង q

ដើម្បីឱ្យបន្ទាត់ $y=x+2$ ជាអាស៊ីមតូតទ្រេតនៃក្រាប (c) តាង f លុះត្រាតែ :

$\lim\limits_{x\to\infty}\left[f(x)-(x+2)\right]=0$ ។

ដោយ $f(x)-(x+2)=\dfrac{x^3+px^2+q}{x^2-2x+2}-(x+1)=\dfrac{(p+1)x^2+(q-2)}{x^2-2x+2}$

គេបាន $\lim\limits_{x\to\infty}\left[f(x)-(x+2)\right]=\lim\limits_{x\to\infty}\dfrac{(p+1)x^2+(q-2)}{x^2-2x+2}=p+1=0$

នាំឱ្យ $p=-1$ ។អនុគមន៍ អាចសរសេរ $f(x)=\dfrac{x^3-x^2+q}{x^2-2x+2}$ ។

ម្យ៉ាងទៀតដោយ ខ្សែកោង (c) តាងអនុគមន៍ f កាត់តាមចំនុច

A(2,4) នោះកូអរដោនេនៃចំនុច A ត្រូវផ្ទៀងផ្ទាត់សមីការ (c) ។

គេបាន $f(2)=\dfrac{8-4+q}{4-4+2}=\dfrac{4+q}{2}=4$ នាំឱ្យ $q=4$ ។

ដូចនេះ $\boxed{p=-1\ ,\ q=4}$ ។

លំហាត់ទី៦

ចូរគណនាលីមីតខាងក្រោម :

ក. $\lim\limits_{x\to 0}\dfrac{1-(1+x)(1+2x)(1+3x)}{x}$

ខ. $\lim\limits_{x\to 0}\dfrac{1-(1+x)(1+2x)(1+3x)......(1+nx)}{x}, n\in IN^*$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម

ក. $\lim\limits_{x\to 0}\dfrac{1-(1+x)(1+2x)(1+3x)}{x}$

$=\lim\limits_{x\to 0}\dfrac{1-(1+x)+(1+x)[1-(1+2x)]+(1+x)(1+2x)[1-(1+3x)]}{x}$

$=\lim\limits_{x\to 0}\dfrac{-x-2x(1+x)-3x(1+x)(1+2x)}{x}$

$=-\lim\limits_{x\to 0}\left[1+2(1+x)+3(1+x)(1+2x)\right]=-(1+2+3)=-6$

ដូចនេះ $\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)}{x}=-6$ ។

ខ. $\lim_{x\to 0}\frac{1-(1+x)(1+2x)(1+3x)......(1+nx)}{x}, n\in IN^*$

$=\lim_{x\to 0}\frac{1-(1+x)+(1+x)[1-(1+2x)]+...+(1+x)(1+2x)...(1+(n-1)x)[1-(1+nx)]}{x}$

$=\lim_{x\to 0}\frac{-x-2x(1+x)-.....-nx(1+x)(1+2x)....(1+(n-1)x)}{x}$

$=-\lim_{x\to 0}[1+2(1+x)+3(1+x)(1+2x)+....+n(1+x)(1+2x)...(1+(n-1)x]$

$=-(1+2+3+.....+n)=-\frac{n(n+1)}{2}$

ដូចនេះ $\lim_{x\to 1}\frac{1-(1+x)(1+2x)(1+3x)....(1+nx)}{x}=-\frac{n(n+1)}{2}$ ។

លំហាត់ទី៧

ចូរគណនាលីមីត :

$$\lim_{x\to\infty}\frac{2x}{1+x^2}.\tan(\frac{\pi x+4}{2x+3})$$

ដំណោះស្រាយ

គណនាលីមីត :

$$\lim_{x\to\infty}\frac{2x}{1+x^2}.\tan(\frac{\pi x+4}{2x+3})$$

គេមាន

$$\frac{\pi x+4}{2x+3}=\frac{1}{2}\frac{2\pi x+8}{2x+3}=\frac{1}{2}\frac{\pi(2x+3)+(8-3\pi)}{2x+3}=\frac{\pi}{2}+\frac{8-3\pi}{2(2x+3)}$$

គេបាន $L=\lim_{x\to\infty}\frac{2x}{1+x^2}.\tan(\frac{\pi x+4}{2x+3})=\lim_{x\to\infty}\frac{2x}{1+x^2}\tan\left[\frac{\pi}{2}+\frac{8-3\pi}{2(2x+3)}\right]$

$$=-\lim_{x\to\infty}\frac{2x}{1+x^2}.\cot\frac{8-3\pi}{2(2x+3)}$$

ពីព្រោះ $\tan\left[\frac{\pi}{2}+\frac{8-3\pi}{2(2x+3)}\right]=-\cot\frac{8-3\pi}{2(2x+3)}$ ។

តាង $y=\frac{8-3\pi}{2(2x+3)}$ កាលណា $x\to\infty$ នោះ $y\to 0$ គេបាន

$$L=-\lim_{\substack{x\to\infty\\ y\to 0}}\frac{2x}{1+x^2}.\cot y=-\lim_{\substack{x\to\infty\\ y\to 0}}\frac{2x}{(1+x^2)}.\frac{1}{y}.\frac{y}{\tan y}$$

$$=-\lim_{\substack{x\to\infty\\ y\to 0}}\frac{2x}{1+x^2}.\frac{2(2x+3)}{8-3\pi}.\frac{y}{\tan y}$$

$$=-\frac{4}{8-3\pi}\lim_{\substack{x\to\infty\\ y\to 0}}\frac{2x^2+3x}{1+x^2}.\frac{y}{\tan y}$$

$$=-\frac{8}{8-3\pi}=\frac{8}{3\pi-8}$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\frac{2x}{1+x^2}.\tan(\frac{\pi x+4}{2x+3})=\frac{8}{3\pi-8}}$ ។

លំហាត់ទី៨

ចូរគណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

ក. $\lim\limits_{x\to+\infty}\left[\ln(2x+1)-\ln(x+2)\right]$

ខ. $\lim\limits_{x\to+\infty}\left[\ln\left(\frac{2x^3+1}{6x^5+4}\right)+\ln\left(\frac{9x^5+5}{4x^3+x}\right)\right]$

គ. $\lim\limits_{x\to+\infty}\frac{6e^x+5}{2e^x+3}$

ឃ. $\lim\limits_{x\to-\infty}\frac{4e^{-2x}+9e^{2x}}{2e^{-2x}+3e^{2x}}$

ង. $\lim\limits_{x\to+\infty}\left[(2x+3)(e^{\frac{2}{x+1}}-1)\right]$

ច. $\lim\limits_{x\to+\infty}\frac{3^x+4^x}{2^x+4^x}$

ឆ. $\lim\limits_{x\to+\infty}\frac{3^x-5^x}{3^{x+1}+5^{x+1}}$

ជ. $\lim\limits_{x\to+\infty}\left[2x+1-2\ln(2e^x+1)\right]$

ឈ. $\lim\limits_{x\to+\infty}\left[\ln(12x^6+1)-3\ln(2x^2+5)\right]$

ញ. $\lim\limits_{x\to+\infty}\left[\,x+1-\ln(\,2e^x+1)\right]$

ដំណោះស្រាយ

គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

ក. $\lim\limits_{x\to+\infty}\left[\ln(2x+1)-\ln(x+2)\right]$

$$=\lim_{x\to+\infty}\left[\ln\left(\frac{2x+1}{x+2}\right)\right]=\ln\left[\lim_{x\to+\infty}\frac{2x+1}{x+2}\right]=\ln 2$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left[\ln(2x+1)-\ln(x+2)\right]=2}$

ខ. $\lim_{x\to+\infty}\left[\ln(\frac{2x^3+1}{6x^5+4})+\ln(\frac{9x^5+5}{4x^3+x}\right]$

$=\lim_{x\to+\infty}\left[\ln\left(\frac{2x^3+1}{6x^5+4}.\frac{9x^5+5}{4x^3+x}\right)\right]$

$=\lim_{x\to+\infty}\left[\ln\frac{x^3(2+\frac{1}{x^3})}{x^5(6+\frac{4}{x^5})}.\frac{x^5(9+\frac{5}{x^5})}{x^3(4+\frac{1}{x^2})}\right]$

$=\lim_{x\to+\infty}\left[\ln\frac{2+\frac{1}{x^3}}{6+\frac{4}{x^5}}.\frac{9+\frac{5}{x^5}}{4+\frac{1}{x^2}}\right]=\ln\left(\frac{2}{6}.\frac{9}{4}\right)=\ln\frac{3}{4}$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\left[\ln(\frac{2x^3+1}{6x^5+4})+\ln(\frac{9x^5+5}{4x^3+x}\right]=\ln\left(\frac{3}{4}\right)}$ ។

គ. $\lim_{x\to+\infty}\frac{6e^x+5}{2e^x+3}$

$=\lim_{x\to+\infty}\frac{e^x(6+\frac{5}{e^x})}{e^x(2+\frac{3}{e^x})}=\lim_{x\to+\infty}\frac{6+\frac{5}{e^x}}{2+\frac{3}{e^x}}=\frac{6}{2}=3$

ព្រោះ $\lim_{x\to+\infty}\frac{1}{e^x}=0$ ។

ដូចនេះ $\boxed{\lim_{x\to+\infty}\frac{6e^x+5}{2e^x+3}=3}$ ។

ឃ. $\lim_{x\to-\infty} \frac{4e^{-2x}+9e^{2x}}{2e^{-2x}+3e^{2x}}$

$$\lim_{x\to-\infty} \frac{\frac{4}{e^{2x}}+9e^{2x}}{\frac{2}{e^{2x}}+3e^{2x}} = \lim_{x\to-\infty} \frac{4+9e^{4x}}{2+3e^{4}x} = \frac{4}{2} = 2$$

ព្រោះ $\lim_{x\to-\infty} e^{4x} = 0$ ។

ដូចនេះ $\boxed{\lim_{x\to-\infty} \frac{4e^{-2x}+9e^{2x}}{2e^{-2x}+3e^{2x}} = 2}$ ។

ង. $\lim_{x\to+\infty}\left[(2x+3)(e^{\frac{2}{x+1}}-1)\right]$

តាង $y=\frac{2}{x+1}$ បើ $x\to+\infty$ នោះ $y\to 0$

$$= \lim_{\substack{x\to+\infty\\ y\to 0}} \left[(2x+3)(e^{y}-1)\right]$$

$$= \lim_{\substack{x\to+\infty\\ y\to 0}} \left[(2x+3).y.\frac{e^{y}-1}{y}\right] = \lim_{\substack{x\to+\infty\\ y\to 0}} \left[(2x+3)\frac{2}{x+1}.\frac{e^{y}-1}{y}\right]$$

$$= \lim_{x\to+\infty} \frac{4x+6}{x+1}.\lim_{y\to 0}\frac{e^{y}-1}{y} = 4.1 = 4$$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\left[(2x+3)(e^{\frac{2}{x+1}}-1)\right] = 4}$ ។

ច. $\lim\limits_{x\to+\infty}\dfrac{3^x+4^x}{2^x+4^x}$

$$=\lim_{x\to+\infty}\frac{4^x(\frac{3^x}{4^x}+1)}{4^x(\frac{2^x}{4^x}+1)}=\lim_{x\to+\infty}\frac{(\frac{3}{4})^x+1}{(\frac{1}{2})^x+1}=1$$

ព្រោះ $\lim\limits_{x\to+\infty}(\frac{3}{4})^x=0$, $\lim\limits_{x\to+\infty}(\frac{1}{2})^x=0$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\dfrac{3^x+4^x}{2^x+4^x}=1}$ ។

ឆ. $\lim\limits_{x\to+\infty}\dfrac{3^x-5^x}{3^{x+1}+5^{x+1}}$

$$=\lim_{x\to+\infty}\frac{5^x(\frac{3^x}{5^x}-1)}{5^{x+1}(\frac{3^{x+1}}{5^{x+1}}+1)}$$

$$=\lim_{x\to+\infty}\frac{1}{5}.\frac{(\frac{3}{5})^x-1}{(\frac{3}{5})^{x+1}-1}=\frac{1}{5}$$

ព្រោះ $\lim\limits_{x\to+\infty}\left(\frac{3}{5}\right)^x=0$ ។

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\dfrac{3^x-5^x}{3^{x+1}+5^{x+1}}=\dfrac{1}{5}}$ ។

ជ. $\lim_{x\to+\infty}\left[2x+1-2\ln(2e^x+1)\right]$

$= \lim_{x\to+\infty}\left[\ln e^{2x+1}-\ln(2e^x+1)^2\right]$

$= \lim_{x\to+\infty}\ln\left[\frac{e^{2x+1}}{(2e^x+1)^2}\right] = \lim_{x\to+\infty}\ln\left[\frac{e^{2x}.e}{e^{2x}(2+\frac{1}{e^x})^2}\right]$

$= \lim_{x\to+\infty}\ln\left[\frac{e}{(2+\frac{1}{e^x})^2}\right] = \ln(\frac{e}{4})$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\left[2x+1-2\ln(2e^x+1)\right] = \ln(\frac{e}{4})}$ ។

ឈ. $\lim_{x\to+\infty}\left[\ln(12x^6+1)-3\ln(2x^2+5)\right]$

$= \lim_{x\to+\infty}\left[\ln(12x^6+1)-\ln(2x^2+5)^3\right]$

$= \lim_{x\to+\infty}\left[\ln\frac{12x^6+1}{(2x^2+5)^3}\right] = \lim_{x\to+\infty}[\ln\frac{x^6(12+\frac{1}{x^6})}{x^6(2+\frac{5}{x^2})^3}]$

$= \lim_{x\to+\infty}[\ln(\frac{12+\frac{1}{x^6}}{(2+\frac{5}{x^2})^3}] = \ln(\frac{12}{2}) = \ln 6$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\left[\ln(12x^6+1)-3\ln(2x^2+5)\right] = \ln 6}$ ។

លំហាត់ទី៩

ចូរគណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{e^{x}-e^{-x}}{\sin 2x}$

ខ. $\lim\limits_{x\to 0}\dfrac{e^{ax}-e^{bx}}{x}$,$a,b\in IR^*$

គ. $\lim\limits_{x\to 0}\dfrac{2e^{x}+3e^{2x}-5}{e^{3x}-1}$

ឃ. $\lim\limits_{x\to 0}\dfrac{e^{x}+e^{2x}+...+e^{nx}-n}{x}$

ង. $\lim\limits_{x\to 0}\dfrac{(e^{x}-1)(e^{2x}-1)...(e^{nx}-1)}{x^{n}}$

ច. $\lim\limits_{x\to 0}\dfrac{e^{-x^2}-\cos 2x}{x^2}$

ឆ. $\lim\limits_{x\to 0}\dfrac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}$

ជ. $\lim\limits_{x\to 0}\dfrac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}$

ឈ. $\lim\limits_{x\to 0}\dfrac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}$

ញ. $\lim\limits_{x\to 0}\dfrac{\sqrt{e^{x^2}+3}-2\cos 4x}{x\sin x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to 0}\dfrac{e^{x}-e^{-x}}{\sin 2x}$

$$=\lim_{x\to 0}\frac{e^{x}-\frac{1}{e^{x}}}{\sin 2x}=\lim_{x\to 0}\frac{e^{2x}-1}{e^{x}\sin 2x}$$

$$=\lim_{x\to 0}\frac{e^{2x}-1}{2x}.\frac{2x}{\sin 2x}.\frac{1}{e^{x}}=1$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^{x}-e^{-x}}{\sin 2x}=1}$ ។

ខ. $\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}\ ,a,b\in IR^*$

$$=\lim_{x\to 0}\frac{(e^{ax}-1)-(e^{bx}-1)}{x}$$

$$=\lim_{x\to 0}\frac{e^{ax}-1}{ax}.a-\lim_{x\to 0}\frac{e^{bx}-1}{bx}.b=a-b$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^{ax}-e^{bx}}{x}=a-b}$

គ. $\lim_{x\to 0}\frac{2e^{x}+3e^{2x}-5}{e^{3x}-1}$

$$=\lim_{x\to 0}\frac{2(e^{x}-1)+3(e^{2x}-1)}{(e^{3x}-1)}$$

$$=\lim_{x\to 0}\frac{2.\frac{e^{x}-1}{x}+6.\frac{e^{2x}-1}{2x}}{3.\frac{e^{3x}-1}{3x}}=\frac{2+6}{3}=\frac{8}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{2e^{x}+3e^{2x}-5}{e^{3x}-1}=\frac{8}{3}}$ ។

ឃ. $\lim_{x\to 0}\frac{e^x+e^{2x}+...+e^{nx}-n}{x}$

$=\lim_{x\to 0}\frac{(e^x-1)+(e^{2x}-1)+........+(e^{nx}-1)}{x}$

$=\lim_{x\to 0}\left[\frac{e^x-1}{x}+2.\frac{e^{2x}-1}{2x}+.....+n.\frac{e^{nx}-1}{nx}\right]$

$=1+2+..........+n=\frac{n(n+1)}{2}$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^x+e^{2x}+...+e^{nx}-n}{x}=\frac{n(n+1)}{2}}$ ។

ង. $\lim_{x\to 0}\frac{(e^x-1)(e^{2x}-1)...(e^{nx}-1)}{x^n}$

$=\lim_{x\to 0}\frac{e^x-1}{x}.2\frac{e^{2x}-1}{2x}......n\frac{e^{nx}-1}{nx}=1.2.3......n=n!$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{(e^x-1)(e^{2x}-1)...(e^{nx}-1)}{x^n}=\frac{n(n+1)}{2}}$ ។

ច. $\lim_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}$

$=\lim_{x\to 0}\frac{e^{-x^2}-(1-2\sin^2 x)}{x^2}$

$=\lim_{x\to 0}\frac{e^{-x^2}-1+2\sin^2 x}{x^2}=-\lim_{x\to 0}\frac{e^{-x^2}-1}{-x^2}+2\lim_{x\to 0}\frac{\sin^2 x}{x}=-1+2=1$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}=1}$ ។

ឆ. $\lim\limits_{x\to 0}\dfrac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}$

$=\lim\limits_{x\to 0}\dfrac{(e^{-2\sin x}-1)-(e^{\tan 3x}-1)}{x(x^2+1)}$

$=\lim\limits_{x\to 0}\dfrac{e^{-2\sin x}-1}{-2\sin x}.\dfrac{\sin x}{x}.\dfrac{-2}{x^2+1}-\lim\limits_{x\to 0}\dfrac{e^{\tan 3x}-1}{\tan 3x}.\dfrac{\tan 3x}{3x}.\dfrac{3}{x^2+1}$

$=-2-3=-5$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\dfrac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}=-5}$ ។

ជ. $\lim\limits_{x\to 0}\dfrac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}$

$=\lim\limits_{x\to 0}\dfrac{x(e^{-2x^2}-1)+\cos x\tan x-\tan x}{x^3}$

$=\lim\limits_{x\to 0}\dfrac{e^{-2x^2}-1}{x^2}+\lim\limits_{x\to 0}\dfrac{\tan x(\cos x-1)}{x^3}$

$=-2\lim\limits_{x\to 0}\dfrac{e^{-2x^2}-1}{-2x^2}-2\lim\limits_{x\to 0}\dfrac{\tan x}{x}.\dfrac{\sin^2\frac{x}{2}}{x^2}=-2-2(\frac{1}{2})^2=-\dfrac{5}{2}$

ដូចនេះ $\lim\limits_{x\to 0}\dfrac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}=-\dfrac{5}{2}$ ។

ឈ. $\lim\limits_{x\to 0}\dfrac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}$

$$= \lim_{x\to 0} \frac{e^{3\sin^2 x} - (1-2\sin^2\frac{x}{2})(1-2\sin^2\frac{3x}{2})}{-e^{2x^2} + 1 - 2\sin^2 x}$$

$$= \lim_{x\to 0} \frac{e^{3\sin^2 x} - 1 + 2\sin^2\frac{3x}{2} + 2\sin^2\frac{x}{2} - 4\sin^2\frac{x}{2}\sin^2\frac{3x}{2}}{-(e^{2x^2} - 1 + 2\sin^2 x)}$$

$$= -\lim_{x\to 0} \frac{3.\frac{e^{3\sin^2 x}-1}{3\sin^2 x}.\frac{\sin^2 x}{x^2} + 2\frac{\sin^2\frac{3x}{2}}{x^2} + 2\frac{\sin^2\frac{x}{2}}{x^2} - 4\frac{\sin^2\frac{x}{2}}{x^2}.\sin^2\frac{3x}{2}}{2.\frac{e^{2x^2}-1}{2x^2} + 2.\frac{\sin^2 x}{x^2}}$$

$$= -\frac{3 + 2.\frac{9}{4} - 2.\frac{1}{4} - 0}{2+2} = \frac{3+4}{4} = \frac{7}{4}$$

ដូចនេះ $\lim_{x\to 0} \frac{e^{3\sin^2 x} - \cos x\cos 3x}{-e^{2x^2} + \cos 2x} = \frac{7}{4}$ ។

លំហាត់ទី១០

គណនាលីមីត ៖

ក. $\lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+5}$

ខ. $\lim\limits_{n\to+\infty}\left(\frac{n-1}{n+1}\right)^{n+2}$

ដំណោះស្រាយ

គណនាលីមីត ៖

ក. $\lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+5}$

តាង $x=\frac{1}{n}$ កាលណា $n\to+\infty$ នោះ $x\to 0$

គេបាន

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+5}=\lim_{x\to 0}(1+x)^{\frac{1}{x}+5}$$

$$=\lim_{x\to 0}(1+x)^{\frac{1}{x}}.(1+x)^{5}=e$$

ដូចនេះ $\boxed{\lim\limits_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+5}=e}$ ។

ខ. $\lim\limits_{n\to+\infty}\left(\frac{n-1}{n+1}\right)^{n+2}$

តាង $1+x=\frac{n-1}{n+1}$ នាំឱ្យ $x=-\frac{2}{n+1}$ និង $n=-\frac{2+x}{x}$

កាលណា $n\to+\infty$ នោះ $x\to 0$

គេបាន

$$\lim_{n\to+\infty}\left(\frac{n-1}{n+1}\right)^{n+2}=\lim_{x\to0}(1+x)^{-\frac{2+x}{x}+2}$$

$$=\lim_{x\to0}(1+x)^{-\frac{2}{x}+1}$$

$$=\lim_{x\to0}\left[(1+x)^{\frac{1}{x}}\right]^{-2}.(1+x)=e^{-2}=\frac{1}{e^2}$$

ដូចនេះ $\boxed{\lim\limits_{n\to+\infty}\left(\frac{n-1}{n+1}\right)^{n+2}=\frac{1}{e^2}}$ ។

លំហាត់ទី១១

គណនាលីមីតខាងក្រោម ៖

ក. $\lim\limits_{x\to0}\frac{e^x+2\sin x-1}{x}$

ខ. $\lim\limits_{n\to+\infty}\left(\frac{2n}{2n-1}\right)^{n+1}$

ដំណោះស្រាយ

គណនាលីមីត ៖

ក. $\lim_{x\to 0}\frac{e^x+2\sin x-1}{x}$

$$=\lim_{x\to 0}\left(\frac{e^x-1}{x}+2\frac{\sin x}{x}\right)$$

$$=\lim_{x\to 0}\frac{e^x-1}{x}+2\lim_{x\to 0}\frac{\sin x}{x}=1+2=3$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{e^x+2\sin x-1}{x}=3}$ ។

ខ. $\lim_{n\to+\infty}\left(\frac{2n}{2n-1}\right)^{n+1}$

$$=\lim_{n\to+\infty}\left(1+\frac{1}{2n-1}\right)^{n+1}$$

$$=\lim_{n\to+\infty}\left(\left(1+\frac{1}{2n-1}\right)^{2n-1}\right)^{\frac{n+1}{2n-1}}=e^{\frac{1}{2}}=\sqrt{e}$$

ដូចនេះ $\boxed{\lim_{n\to+\infty}\left(\frac{2n}{2n-1}\right)^{n+1}=\sqrt{e}}$ ។

លំហាត់ទី១២

ចូរគណនាលីមីត

$$L_n = \lim_{x\to 2} \frac{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}-2}{x-2} \text{ (មាន n ឫសការេ)}$$

ដំណោះស្រាយ

គណនាលីមីត :

$$L_n = \lim_{x\to 2} \frac{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}-2}{x-2} \text{ (មាន n ឫសការេ)}$$

យើងបាន :

$$L_n = \lim_{x\to 2} \frac{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}-2}{x-2} \times \frac{\sqrt{2+\sqrt{2+\sqrt{2+...+\sqrt{2+x}}}}+2}{\sqrt{2+\sqrt{2+\sqrt{2+...+\sqrt{2+x}}}}+2}$$

$$L_n = \lim_{x\to 2} \frac{2+\sqrt{2+\sqrt{2+.....+\sqrt{2+x}}}-4}{x-2} \times \frac{1}{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}+2}$$

$$L_n = \lim_{x\to 2} \frac{\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}-2}{x-2} \times \lim_{x\to 2} \frac{1}{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}+2}$$

$$L_n = L_{n-1} \times \frac{1}{4} = \frac{1}{4} L_{n-1}$$

តាមទំនាក់ទំនងនេះបញ្ជាក់ថា $(L_n), n \in IN^*$ ជាស្វ៊ីតធរណីមាត្រមានរេសុង

$q = \frac{1}{4}$ និងតួទីមួយ

$$L_1 = \lim_{x\to 2} \frac{\sqrt{2+x}-2}{x-2} = \lim_{x\to 2} \frac{2+x-4}{(x-2)(\sqrt{2+x}+2)} = \lim_{x\to 2} \frac{x-2}{(x-2)(\sqrt{2+x}+2)} = \frac{1}{4}$$

តាមរូបមន្ត $L_n = L_1 \times q^{n-1} = \frac{1}{4}.\left(\frac{1}{4}\right)^{n-1} = \frac{1}{4^n}$ ។

ដូចនេះ $L_n = \lim_{x\to 2} \frac{\sqrt{2+\sqrt{2+\sqrt{2+....+\sqrt{2+x}}}}-2}{x-2} = \frac{1}{4^n}$ ។

លំហាត់ទី១៣

គេឱ្យអាំងតេក្រាល $I_0 = \int_0^1 \frac{dt}{1+t+t^2}$

និង $I_n = \int_0^1 \frac{t^n}{1+t+t^2}.dt$, $(n \in IN)$

ក. ចូរគណនាតម្លៃនៃ I_0 រួច ស្រាយថា (I_n) ជាស្វ៊ីតចុះ ។

ខ. ស្រាយបញ្ជាក់ថា $I_n + I_{n+1} + I_{n+2} = \frac{1}{n+1}$ ។

គ. ទាញឱ្យបានថា $\frac{1}{3(n+1)} \le I_n \le \frac{1}{3(n-1)}$, $\forall n \ge 2$ ។

ទាញរកលីមីត $\lim_{n\to+\infty} (n\,I_n)$ ។

ដំណោះស្រាយ

ក. គណនាតម្លៃនៃ I_0 រួច ស្រាយថា (I_n) ជាស្វ៊ីតចុះ

យើងបាន $I_0 = \int_0^1 \frac{dt}{1+t+t^2} = \int_0^1 \frac{dt}{\frac{3}{4}+(\frac{1}{2}+t)^2}$

តាង $U=\frac{1}{2}+t$ នាំឱ្យ $dU=dt$

ហើយចំពោះ $\forall t\in[0,1]$ នោះ $U\in\left[\frac{1}{2},\frac{3}{2}\right]$

គេបាន $I_0=\int_{\frac{1}{2}}^{\frac{3}{2}}\frac{dU}{(\frac{\sqrt{3}}{2})^2+U^2}=\left[\frac{2}{\sqrt{3}}\arctan\left(\frac{2U}{\sqrt{3}}\right)\right]_{\frac{1}{2}}^{\frac{3}{2}}$

$$=\frac{2}{\sqrt{3}}\arctan\sqrt{3}-\frac{2}{\sqrt{3}}\arctan\frac{1}{\sqrt{3}}=\frac{2}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=\frac{\pi}{3\sqrt{3}}$$ ។

ដូចនេះ $I_0=\int_0^1\frac{dt}{1+t+t^2}=\frac{\pi}{3\sqrt{3}}$ ។

ម្យ៉ាងទៀតគេមាន $I_n=\int_0^1\frac{t^n}{1+t+t^2}.dt$ និង $I_{n+1}=\int_0^1\frac{t^{n+1}}{1+t+t^2}.dt$

ចំពោះគ្រប់ $t\in[0,1]$ គេមាន $t^{n+1}\le t^n$

នាំឱ្យ $\frac{t^{n+1}}{1+t+t^2}\le\frac{t^n}{1+t+t^2}$

គេទាញ $\int_0^1\frac{t^{n+1}}{1+t+t^2}.dt\le\int_0^1\frac{t^n}{1+t+t^2}.dt$

ឬ $I_{n+1}\le I_n \ ,\forall n\in IN$ ។

ដូចនេះ (I_n) ជាស្វ៊ីតចុះ ។

ខ. ស្រាយបញ្ជាក់ថា $I_n+I_{n+1}+I_{n+2}=\frac{1}{n+1}$

យើងបាន $I_n+I_{n+1}+I_{n+2}=\int_0^1\frac{t^n dt}{1+t+t^2}+\int_0^1\frac{t^{n+1}dt}{1+t+t^2}+\int_0^1\frac{t^{n+2}dt}{1+t+t^2}$

$$=\int_0^1 \frac{(t^n + t^{n+1} + t^{n+2})dt}{1+t+t^2} = \int_0^1 \frac{t^n(1+t+t^2).dt}{1+t+t^2}$$

$$=\int_0^1 t^n dt = \left[\frac{1}{n+1}t^{n+1}\right]_0^1 = \frac{1}{n+1}$$

ដូចនេះ $I_n + I_{n+1} + I_{n+2} = \frac{1}{n+1}$ ។

គ. ទាញឱ្យបានថា $\frac{1}{3(n+1)} \le I_n \le \frac{1}{3(n-1)}$, $\forall n \ge 2$

យើងមាន (I_n) ជាស្វ៊ីតចុះ ។ តាមលក្ខណៈនៃស្វ៊ីតចុះយើងមាន :

$I_n + I_{n+1} + I_{n+2} \le 3I_n \le I_{n-2} + I_{n-1} + I_n$

ដោយ $I_n + I_{n+1} + I_{n+2} = \frac{1}{n+1}$

នាំឱ្យ $I_{n-2} + I_{n-1} + I_n = \frac{1}{n-1}$

គេទាញ $\frac{1}{n+1} \le 3I_n \le \frac{1}{n-1}$

នាំឱ្យ $\frac{1}{3(n+1)} \le I_n \le \frac{1}{3(n-1)}$, $\forall n \ge 2$ ។

ទាញរកលីមីត $\lim\limits_{n\to+\infty}(n I_n)$

មាន $\frac{1}{3(n+1)} \le I_n \le \frac{1}{3(n-1)}$, $\forall n \ge 2$

នាំឱ្យ $\frac{n}{3(n+1)} \le nI_n \le \frac{n}{3(n-1)}$

ដូចនេះ $\lim\limits_{n\to+\infty}(nI_n) = \frac{1}{3}$ ។

លំហាត់ទី១៨

គេឱ្យអាំងតេក្រាល $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \cos^3 x.dx$ ដែល $n \in IN$

ក. ចូរគណនា I_n ជាអនុគមន៍នៃ n ។

ខ. ចូរគណនាផលបូក $S_n = \sum_{k=0}^{n}(I_k) = I_0 + I_1 + I_2 + ... + I_n$

ជាអនុគមន៍នៃ n រួចទាញរកតម្លៃនៃលីមីត $\lim_{n\to+\infty} S_n$ ។

ដំណោះស្រាយ

ក. គណនា I_n ជាអនុគមន៍នៃ n

យើងមាន :

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \cos^3 x.dx$$

$$= \int_0^{\frac{\pi}{2}} \sin^n x \cos^2 x.\cos x.dx = \int_0^{\frac{\pi}{2}} \sin^n x(1-\sin^2 x)\cos x.dx$$

$$= \int_0^{\frac{\pi}{2}} (\sin^n x - \sin^{n+2} x)\cos x.dx$$

តាង $U = \sin x$ នាំឱ្យ $dU = \cos x.dx$

ចំពោះ $x \in [0, \frac{\pi}{2}]$ នាំឱ្យ $U \in [0\,,1\,]$

យើងបាន :

$$I_n=\int_0^1 U^n(1-U^2).dU=\int_0^1 U^n.dU-\int_0^1 U^{n+2}.dU$$

$$=\left[\frac{1}{n+1}U^{n+1}\right]_0^1-\left[\frac{1}{n+3}U^{n+3}\right]_0^1$$

$$=\frac{1}{n+1}-\frac{1}{n+3}=\frac{n+3-n-1}{(n+1)(n+3)}=\frac{2}{(n+1)(n+3)}$$

ដូចនេះ $\boxed{I_n=\frac{2}{(n+1)(n+3)}}$ ។

ខ. គណនាផលបូក $S_n=\sum_{k=0}^{n}(I_k)=I_0+I_1+I_2+.........+I_n$

តាមសម្រាយខាងលើយើងមាន :

$$I_n=\frac{2}{(n+1)(n+3)}=\frac{1}{n+1}-\frac{1}{n+3}=\left(\frac{1}{n+1}-\frac{1}{n+2}\right)+\left(\frac{1}{n+2}-\frac{1}{n+3}\right)$$

យើងបាន :

$$S_n=\sum_{k=0}^{n}\left[\left(\frac{1}{k+1}-\frac{1}{k+2}\right)+\left(\frac{1}{k+2}-\frac{1}{k+3}\right)\right]$$

$$=\sum_{k=0}^{n}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)+\sum_{k=0}^{n}\left(\frac{1}{k+2}-\frac{1}{k+3}\right)$$

$$=\left[(1-\frac{1}{2})+(\frac{1}{2}-\frac{1}{3})+....+(\frac{1}{n+1}-\frac{1}{n+2})\right]+\left[(\frac{1}{2}-\frac{1}{3})+(\frac{1}{3}-\frac{1}{4})+....+(\frac{1}{n+2}-\frac{1}{n+3})\right]$$

$$=\left(1-\frac{1}{n+2}\right)+\left(\frac{1}{2}-\frac{1}{n+3}\right)=\frac{3}{2}-\frac{1}{n+2}-\frac{1}{n+3}=\frac{(n+1)(3n+8)}{2(n+2)(n+3)}$$

ដូចនេះ $\boxed{S_n=\frac{(n+1)(3n+8)}{2(n+2)(n+3)}}$ ។

គណនាតម្លៃនៃលីមីត $\lim\limits_{n\to+\infty} S_n$

យើងមាន $S_n = \frac{3}{2} - \frac{1}{n+2} - \frac{1}{n+3}$

យើងបាន $\lim\limits_{n\to+\infty} S_n = \lim\limits_{n\to+\infty}\left(\frac{3}{2} - \frac{1}{n+2} - \frac{1}{n+3}\right) = \frac{3}{2}$ ។

ដូចនេះ $\boxed{\lim\limits_{n\to+\infty} S_n = \frac{3}{2}}$ ។

លំហាត់ទី១៥

គេឱ្យស្វ៊ីតអាំងតេក្រាល $I_n = \int_0^1 x^n \sqrt{1-x^2}\,.dx$ ដែល $n \in \mathbb{N}$ ។

ក. ចូររកទំនាក់ទំនងរវាង I_n និង I_{n-2}

ខ. គណនាផលគុណ $P_n = I_n . I_{n-1}$, $\forall n \geq 1$ ជាអនុគមន៍នៃ n ។

គ. ចូរគណនាផលបូក $S_n = \sum\limits_{k=1}^{n}(P_k) = P_1 + P_2 + \dots\dots\dots + P_n$

ជាអនុគមន៍នៃ n រួចទាញរកតម្លៃនៃលីមីត $\lim\limits_{n\to+\infty} S_n$ ។

ឃ. រករូបមន្តគណនា I_n ជាអនុគមន៍នៃ n ។

ដំណោះស្រាយ

ក. រកទំនាក់ទំនងរវាង I_n និង I_{n-2}

យើងមាន $I_n = \int_0^1 x^n \sqrt{1-x^2}\,.dx$ និង $I_{n-2} = \int_0^1 x^{n-2}.\sqrt{1-x^2}.dx$

តាង $\begin{cases} U=\sqrt{1-x^2} \\ dV=x^n.dx \end{cases}$ នាំឱ្យ $\begin{cases} dU=-\dfrac{x}{\sqrt{1-x^2}}.dx \\ V=\int x^n.dx=\dfrac{1}{n+1}x^{n+1} \end{cases}$

យើងបាន $I_n=\left[\dfrac{1}{n+1}x^{n+1}.\sqrt{1-x^2}\right]_0^1+\dfrac{1}{n+1}\int_0^1\dfrac{x^{n+2}}{\sqrt{1-x^2}}.dx$

$$I_n=\frac{1}{n+1}\int_0^1\frac{x^n-(x^n-x^{n+2})}{\sqrt{1-x^2}}.dx=\frac{1}{n+1}\int_0^1\frac{x^n-x^n(1-x^2)}{\sqrt{1-x^2}}.dx$$

$$I_n=\frac{1}{n+1}\int_0^1\frac{x^n.dx}{\sqrt{1-x^2}}-\frac{1}{n+1}\int_0^1 x^n\sqrt{1-x^2}.dx$$

$$I_n=\frac{1}{n+1}\int_0^1 x^{n-1}.\frac{xdx}{\sqrt{1-x^2}}-\frac{1}{n+1}I_n$$

តាង $\begin{cases} U=x^{n-1} \\ dV=\dfrac{xdx}{\sqrt{1-x^2}} \end{cases}$ នាំឱ្យ $\begin{cases} dU=(n-1)x^{n-2}.dx \\ V=-\sqrt{1-x^2} \end{cases}$

យើងបាន $I_n=\dfrac{1}{n+1}\left\{\left[-x^{n-1}\sqrt{1-x^2}\right]_0^1+(n-1)\int_0^1 x^{n-2}\sqrt{1-x^2}.dx\right\}-\dfrac{1}{n+1}I_n$

គេទាញ $(n+1)I_n=(n-1)I_{n-2}-I_n$ នាំឱ្យ $I_n=\dfrac{n-1}{n+2}I_{n-2}$

ដូចនេះ ទំនាក់ទំនងរវាង I_n និង I_{n-2} គឺ $\boxed{I_n=\dfrac{n-1}{n+2}I_{n-2}\,,\forall n\ge 2}$ ។

ខ. គណនាផលគុណ $P_n=I_n.I_{n-1}\ ,\forall n\ge 1$ ជាអនុគមន៍នៃ n

យើងមាន $P_n=I_n.I_{n-1}$ នាំឱ្យ $P_{n+1}=I_{n+1}.I_n$

ដោយ $I_n = \frac{n-1}{n+2} I_{n-2}$ នាំឱ្យ $I_{n+1} = \frac{n}{n+3}.I_{n-1}$

គេទាញ $P_{n+1} = \frac{n}{n+3} I_{n-1}.I_n = \frac{n}{n+3} P_n$ ព្រោះ $P_n = I_n.I_{n-1}$

នាំអោយ $\frac{P_{n+1}}{P_n} = \frac{n}{n+3} = \frac{n}{n+1}.\frac{n+1}{n+2}.\frac{n+2}{n+3}$ ។

យើងបាន $\prod_{k=1}^{(n-1)} \left(\frac{P_{k+1}}{P_k}\right) = \prod_{k=1}^{(n-1)} \left(\frac{k}{k+1}.\frac{k+1}{k+2}.\frac{k+2}{k+3}\right)$

$$\prod_{k=1}^{(n-1)} \left(\frac{P_{k+1}}{P_k}\right) = \prod_{k=1}^{(n-1)} \left(\frac{k}{k+1}\right) \times \prod_{k=1}^{(n-1)} \left(\frac{k+1}{k+2}\right) \times \prod_{k=1}^{(n-1)} \left(\frac{k+2}{k+3}\right)$$

$$\frac{P_2}{P_1}.\frac{P_3}{P_2}....\frac{P_n}{P_{n-1}} = \left(\frac{1}{2}.\frac{2}{3}....\frac{n-1}{n}\right) \times \left(\frac{2}{3}.\frac{3}{4}....\frac{n}{n+1}\right) \times \left(\frac{3}{4}.\frac{4}{5}....\frac{n+1}{n+2}\right)$$

$$\frac{P_n}{P_1} = \frac{1}{n}.\frac{2}{n+1}.\frac{3}{n+2}$$

នាំឱ្យគេទាញ $P_n = \frac{6}{n(n+1)(n+2)}.P_1 = \frac{6}{n(n+1)(n+2)}.I_0.I_1$

(ព្រោះ $P_1 = I_0.I_1$)

ដោយ $I_0 = \int_0^1 \sqrt{1-x^2}.dx = \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\arcsin x\right]_0^1 = \frac{\pi}{4}$

និង $I_1 = \int_0^1 x\sqrt{1-x^2}.dx = \left[-\frac{1}{3}(1-x^2)^{\frac{3}{2}}\right]_0^1 = \frac{1}{3}$

គេបាន $P_n = \frac{6}{n(n+1)(n+2)}.\frac{\pi}{4}.\frac{1}{3} = \frac{\pi}{2}.\frac{1}{n(n+1)(n+2)}$

ដូចនេះ $\boxed{P_n = \frac{\pi}{2}.\frac{1}{n(n+1)(n+2)}}$ ។

គ. គណនាផលបូក $S_n = \sum_{k=1}^{n}(P_k) = P_1 + P_2 + ... + P_n$

យើងមាន :

$$P_n = \frac{\pi}{2}.\frac{1}{n(n+1)(n+2)}$$

$$= \frac{\pi}{4}\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right]$$

យើងបាន $S_n = \frac{\pi}{4}\sum_{k=1}^{n}\left[\frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)}\right]$

$$= \frac{\pi}{4}\left[(\frac{1}{1.2} - \frac{1}{2.3}) + (\frac{1}{2.3} - \frac{1}{3.4}) + ... + \left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)\right]$$

$$= \frac{\pi}{4}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right] = \frac{\pi}{8}.\frac{n(n+3)}{(n+1)(n+2)}$$

ដូចនេះ $\boxed{S_n = \frac{\pi}{8}.\frac{n(n+3)}{(n+1)(n+2)}}$ និង $\boxed{\lim_{n\to+\infty} S_n = \frac{\pi}{8}}$ ។

ឃ. រករូបមន្តគណនា I_n ជាអនុគមន៍នៃ n

តាមសម្រាយខាងលើយើងមាន $I_n = \frac{n-1}{n+2}I_{n-2} , \forall n \geq 2$

-ករណី $n = 2p+1$ (ចំនួនសេស)

យើងបាន $I_{2p+1} = \frac{2p}{2p+3}.I_{2p-1}$ ឬ $\frac{I_{2p+1}}{I_{2p-1}} = \frac{2p}{2p+3}$

គេទាញ $\frac{I_3}{I_1}.\frac{I_5}{I_3}.\frac{I_7}{I_5}.....\frac{I_{2p+1}}{I_{2p-1}} = \frac{2}{5}.\frac{4}{7}.\frac{6}{9}.....\frac{2p}{2p+3}$

$$\frac{I_{2p+1}}{I_1}=\frac{2}{5}.\frac{4}{7}.\frac{6}{9}.....\frac{2p}{2p+3}$$

នាំឱ្យ $I_{2p+1}=\frac{2.4.6....2p}{5.7.9....(2p+3)}.I_1$ ដោយ $I_1=\frac{1}{3}$

ដូចនេះ $\boxed{I_{2p+1}=\frac{2.4.6....2p}{5.7.9....(2p+3)}.\frac{1}{3}}$ ។

-ករណី $n=2p$ (ចំនួនគូ)

យើងបាន $I_{2p}=\frac{2p-1}{2p+2}.I_{2p-2}$ ឬ $\frac{I_{2p}}{I_{2p-2}}=\frac{2p-1}{2p+2}$

គេទាញ $\frac{I_2}{I_0}.\frac{I_4}{I_2}.\frac{I_6}{I_4}.....\frac{I_{2p}}{I_{2p-2}}=\frac{1}{4}.\frac{3}{6}.\frac{5}{8}.....\frac{2p-1}{2p+2}$

$$\frac{I_{2p}}{I_0}=\frac{1}{4}.\frac{3}{6}.\frac{5}{8}.....\frac{2p-1}{2p+2}$$

នាំឱ្យ $I_{2p+1}=\frac{1.3.5....(2p-1)}{4.6.8....(2p+2)}.I_0$ ដោយ $I_0=\frac{\pi}{4}$

ដូចនេះ $\boxed{I_{2p+1}=\frac{1.3.5....(2p-1)}{4.6.8....(2p+2)}.\frac{\pi}{4}}$ ។

លំហាត់ទី១៦

គេឱ្យស្វ៊ីត (u_n) នៃចំនួនពិតកំនត់លើ IN* ដោយ

$$u_n = \frac{1}{n(n+1)(n+2)}$$ ។

ក.កំនត់ចំនួនពិត A, B, C ដើម្បីអោយ

$$\frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$$ ។

ខ.តាង $S_n = u_1 + u_2 + u_3 + ... + u_n$ ។គណនា $\lim_{n \to +\infty} S_n$

គ.ចំពោះគ្រប់ $n \in IN^*$ គេតាង $V_n = u_n - \int_n^{n+1} g(x).dx$ ដែល g

ជាអនុគមន៍កំនត់ដោយ $g(x) = \frac{1}{x(x+1)(x+2)}$

និង $S'_n = V_1 + V_2 + V_3 + ... + V_n$ ។

បង្ហាញថា $S'_n = S_n - \int_1^{n+1} g(x).dx$ ហើយទាញរក S'_n និង $\lim_{n \to +\infty} S'_n$

ដំណោះស្រាយ

ក. កំនត់ចំនួនពិត A, B, C ដែល

$$\frac{1}{n(n+1)(n+2)} = \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n+2}$$

យើងបាន $1 = A(n+1)(n+2) + B(n+2)n + C(n+1)n$

សមមូល $1 = (A+B+C)n^2 + (3A+2B+C)n + 2A$

គេទាញ $\begin{cases} 2A = 1 \\ A + B + C = 0 \\ 3A + 2B + C = 0 \end{cases}$

នាំឱ្យ $A = \frac{1}{2}$, $B = -1$, $C = \frac{1}{2}$ ។

ខ. គណនា $\lim_{n\to+\infty} S_n$

មាន $S_n = u_1 + u_2 + u_3 + ... + u_n = \sum_{k=1}^{n}(u_k)$

$$= \sum_{k=1}^{n}\left(\frac{1}{2k} - \frac{1}{k+1} + \frac{1}{2(k+2)}\right)$$

$$= \sum_{k=1}^{n}\left(\frac{1}{2k} - \frac{1}{2(k+1)}\right) - \sum_{k=1}^{n}\left(\frac{1}{2(k+1)} - \frac{1}{2(k+2)}\right)$$

$$= \frac{1}{2} - \frac{1}{2(n+1)} - \frac{1}{4} + \frac{1}{2(n+2)} = \frac{1}{4} - \frac{1}{2(n+1)} + \frac{1}{2(n+2)}$$

ដូចនេះ $\lim_{n\to+\infty} S_n = \lim_{n\to+\infty}\left[\frac{1}{4} - \frac{1}{2(n+1)} + \frac{1}{2(n+2)}\right] = \frac{1}{4}$ ។

គ.បង្ហាញថា $S'_n = S_n - \int_{1}^{n+1} g(x).dx$ ទាញរក S'_n និង $\lim_{n\to+\infty} S'_n$

ចំពោះគ្រប់ $n \in IN^*$ យើងមាន $V_n = u_n - \int_{n}^{n+1} g(x).dx$

ដែល g ជាអនុគមន៍កំនត់ដោយ $g(x) = \frac{1}{x(x+1)(x+2)}$

និង $S'_n = V_1 + V_2 + V_3 + ... + V_n$

យើងបាន $S'_n = \sum_{k=1}^{n}(V_n) = \sum_{k=1}^{n}\left[u_n - \int_{n}^{n+1} g(x).dx\right]$

$$=\sum_{k=1}^{n}(u_n)-\left[\int_1^2 g(x).dx+\int_2^3 g(x).dx+...+\int_n^{n+1} g(x).dx\right]$$

ដូចនេះ $S'_n = S_n - \int_1^{n+1} g(x).dx$ ។

ម្យ៉ាងទៀតគេមាន $g(x)=\dfrac{1}{x(x+1)(x+2)}$ មានលំនាំដូច

$$u_n=\frac{1}{n(n+1)(n+2)}$$

គេបាន $g(x)=\dfrac{1}{2x}-\dfrac{1}{x+1}+\dfrac{1}{2(x+2)}$

ដោយ $S_n=\dfrac{1}{4}-\dfrac{1}{2(n+1)}+\dfrac{1}{2(n+2)}$ (តាមសម្រាយខាងលើ)

យើងបាន

$$S'_n=\frac{1}{4}-\frac{1}{2(n+1)}+\frac{1}{2(n+2)}-\int_1^{n+1}\left[\frac{1}{2x}-\frac{1}{x+1}+\frac{1}{2(x+2)}\right].dx$$

$$=\frac{1}{4}-\frac{n+2-n-1}{2(n+1)(n+2)}-\left[\frac{1}{2}\ln|x|-\ln|x+1|+\frac{1}{2}\ln|x+2|\right]_1^{n+1}$$

$$=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}-\frac{1}{2}\ln(n+1)+\ln(n+2)-\frac{1}{2}\ln(n+3)+\frac{1}{2}\ln 3-\ln 2$$

$$=\frac{1}{4}+\ln\frac{\sqrt{3}}{2}-\frac{1}{2(n+1)(n+2)}+\ln\left[\frac{n+2}{\sqrt{(n+1)(n+3)}}\right]$$

ដូចនេះ $S'_n=\dfrac{1}{4}+\ln\dfrac{\sqrt{3}}{2}-\dfrac{1}{2(n+1)(n+2)}+\ln\left[\dfrac{n+2}{\sqrt{(n+1)(n+3)}}\right]$ ។

និង $\lim\limits_{n\to+\infty} S'_n=\dfrac{1}{4}+\ln\dfrac{\sqrt{3}}{2}$ ។

លំហាត់ទី១៧

គេមានស៊ីត (I_n) កំនត់ចំពោះគ្រប់ $n \geq 1$ ដោយ

$$I_n = \frac{1}{n!}.\int_0^1 (1-x)^n.e^x.dx$$

ក-ចូរគណនាតួ I_1 ។

ខ-ចូរបញ្ជាក់ I_{n+1} ជាអនុគមន៍នៃ I_n រួចទាញឱ្យបានថា $I_n = e - \sum_{p=0}^{n}\left(\frac{1}{P!}\right)$

គ-ចូររកលីមីត $\lim_{n\to+\infty} I_n$

រួចទាញថា $\lim_{n\to+\infty}\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\frac{1}{n!}\right) = e = 2.71828$

ដំណោះស្រាយ

ក-ចូរគណនាតួ I_1

គេមាន $I_1 = \frac{1}{1!}\int_0^1 (1-x)e^x.dx = \int_0^1 (1-x).e^x.dx$

តាង $\begin{cases} u = 1-x \\ dv = e^x dx \end{cases}$ នាំឱ្យ $\begin{cases} du = -dx \\ v = e^x \end{cases}$

គេបាន $I = \left[(1-x)e^x\right]_0^1 - \int_0^1 e^x(-dx) = -1 + \left[e^x\right]_0^1 = e-2$

ដូចនេះ $\boxed{I = e-2}$ ។

ខ-បញ្ជាក់ I_{n+1} ជាអនុគមន៍នៃ I_n

គេមាន $I_n = \frac{1}{n!}.\int_0^1 (1-x)^n.e^x.dx$

នាំឱ្យ $I_{n+1} = \frac{1}{(n+1)!}.\int_0^1 (1-x)^{n+1}.e^x dx$

តាង $\begin{cases} u = (1-x)^{n+1} \\ dv = e^x dx \end{cases}$ នាំឱ្យ $\begin{cases} du = -(n+1)(1-x)^n \\ v = e^x \end{cases}$

គេបាន $I_{n+1} = \frac{1}{(n+1)!}\left[(1-x)^{n+1}e^x\right]_0^1 + \frac{n+1}{(n+1)!}\int_0^1 (1-x)^n e^x.dx$

$$I_{n+1} = -\frac{1}{(n+1)!} + \frac{1}{n!}\int_0^1 (1-x)^n e^x.dx = -\frac{1}{(n+1)!} + I_n$$

ដូចនេះ $\boxed{I_{n+1} = I_n - \frac{1}{(n+1)!}}$ ។

ទាញឱ្យបានថា $I_n = e - \sum_{p=0}^{n}\left(\frac{1}{P!}\right)$

គេមាន $I_{n+1} = I_n - \frac{1}{(n+1)!}$

ចំពោះ $n = 1 \;:\; I_2 = I_1 - \frac{1}{2!}$

ចំពោះ $n = 2 : I_3 = I_2 - \frac{1}{3!}$

....

ចំពោះ $n = n-1 : I_n = I_{n-1} - \frac{1}{n!}$

ដោយធ្វើផលបូកទំនាក់ទំនងនេះអង្គ និង អង្គ គេបាន :

$I_n = I_1 - \frac{1}{2!} - \frac{1}{3!} - .... - \frac{1}{n!}$ ដោយ $I_1 = e - 2 = e - \frac{1}{0!} - \frac{1}{1!}$

ដូចនេះ $\boxed{I_n = e - \frac{1}{0!} - \frac{1}{1!} - \frac{1}{2!} - .... - \frac{1}{n!} = e - \sum_{p=0}^{n}\left(\frac{1}{p!}\right)}$ ។

គ-ចូររកលីមីត $\lim\limits_{n \to +\infty} I_n$

ចំពោះ $x \in [0, 1]$ គេមាន $1 \le e^x \le e$ និង $(1-x)^n \ge 0$

គេបាន $(1-x)^n \le e^x(1-x)^n \le e(1-x)^n$

នាំឱ្យ $\frac{1}{n!}\int_0^1 (1-x)^n .dx \le \frac{1}{n!}\int_0^1 (1-x)^n e^x .dx \le \frac{e}{n!}\int_0^1 (1-x)^n .dx$

ដោយ $\int_0^1 (1-x)^n .dx = \left[-\frac{1}{n+1}(1-x)^{n+1}\right]_0^1 = \frac{1}{n+1}$

គេទាញបាន $\frac{1}{n!(n+1)} \le I_n \le \frac{e}{n!(n+1)}$ ។

កាលណា $n \to +\infty$ នាំឱ្យ $\frac{1}{n!(n+1)} \to 0$

ដូចនេះ $\boxed{\lim\limits_{n \to +\infty} I_n = 0}$ ។

ទាញថា $\lim\limits_{n \to +\infty}\left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}\right) = e = 2.71828$

គេមាន $I_n = e - \sum_{p=0}^{n}\left(\frac{1}{p!}\right)$ នាំឱ្យ $\sum_{p=0}^{n}\left(\frac{1}{p!}\right) = e - I_n$

គេបាន $\lim\limits_{n\to+\infty}\sum\limits_{p=0}^{n}\left(\frac{1}{p!}\right)=\lim\limits_{n\to+\infty}\left(e-I_n\right)=e$ ព្រោះ $\lim\limits_{n\to+\infty} I_n=0$

ដូចនេះ $$\boxed{\lim_{n\to+\infty}\left(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\frac{1}{n!}\right)=e=2.71828}$$

ជំពូកទី៤

លំហាត់អនុវត្តន៍

១-គណនាលីមីត $A=\lim\limits_{x\to+\infty}\dfrac{\sqrt[4]{x^4+4x^3}+\sqrt[3]{x^3+3x^2}-2\sqrt{x^2+2x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}}$

២-គណនាលីមីតខាងក្រោម :

ក- $\lim\limits_{x\to1}\dfrac{x^n-1}{x-1}\ ,n\in IN$

ខ- $\lim\limits_{x\to1}\dfrac{x+x^2+x^3+...+x^n-n}{x-1}$

គ- $\lim\limits_{x\to0}\dfrac{(x+1)^n-(nx+1)}{x^2}$

ឃ- $\lim\limits_{x\to1}\dfrac{x^{n+1}-(n+1)x+n}{(x-1)^2}$

ង- $\lim\limits_{x\to1}\dfrac{nx^{n+1}-(n+1)x^n+1}{x^{p+1}-x^p-x+1}$

ច- $\lim\limits_{x\to1}\left(\dfrac{1}{1-x}-\dfrac{n}{1-x^n}\right)$

ឆ- $\lim\limits_{x\to1}\left(\dfrac{m}{1-x^m}-\dfrac{n}{1-x^n}\right)$

ជ- $\lim\limits_{x\to0}\dfrac{1-(1+x)(1+2x).....(1+nx)}{x}$

ឈ- $\lim\limits_{x\to1}\dfrac{x+2x^2+...+nx^n-\dfrac{n(n+1)}{2}}{x-1}$

ញ- $\lim\limits_{x\to0}\dfrac{(x+1)(x+2)....(x+n)-n!}{(x+1)(x+2)......(x+p)-p!}$

៣-កំនត់តម្លៃ $m\in IR$ ដើម្បីឱ្យ : $\lim\limits_{x\to2}\dfrac{x^2+mx+4}{x-2}=0$?

៤-កំនត់ a និង b ដើម្បីឱ្យ $\lim\limits_{x\to3}\dfrac{ax^3+bx+6}{x-3}=5$

៥-កំនត់ a និង b ដើម្បីឱ្យ $\lim\limits_{x\to2}\dfrac{x^3+ax^2+bx+4}{x-2}=8$

៦-គណនាលីមីតខាងក្រោម :

ក- $\lim_{x\to 2}\frac{\sqrt{x^2-2}-\sqrt{2}}{x-2}$ ឃ- $\lim_{x\to 2}\frac{x-\sqrt{x+2}}{2-\sqrt{x+2}}$

ខ- $\lim_{x\to 3}\frac{x-3}{\sqrt{2x+3}-\sqrt{x+6}}$ ង- $\lim_{x\to 2}\frac{x-2}{x^3-\sqrt{x+60}}$

គ- $\lim_{x\to 1}\frac{\sqrt{3x+1}-2}{\sqrt{x+3}-2}$

៧- គណនាលីមីតខាងក្រោម :

ក- $\lim_{x\to 2}\frac{\sqrt{2+\sqrt{2+x}}-2}{\sqrt{2+x}-2}$ ង- $\lim_{x\to 0}\frac{\sqrt{x+\sqrt{x+...+\sqrt{x+1}}}-1}{x}$

ខ- $\lim_{x\to 0}\frac{1}{x}(\sqrt{x+\sqrt{x+1}}-1)$ ច- $\lim_{x\to 4}\frac{\sqrt[3]{5x+7}-\sqrt{2x+1}}{\sqrt[4]{3x^2+18}-\sqrt{3x+4}}$

គ- $\lim_{x\to 3}\frac{x-6+\sqrt{x-2}+\sqrt{x+1}}{x-3}$ ឆ- $\lim_{x\to 2}\frac{\sqrt{x^3-\sqrt{x^2+60}}}{\sqrt{x-2}}$

ឃ- $\lim_{x\to 2}\frac{x-2}{2-\sqrt{x+\sqrt{x+2}}}$ ជ- $\lim_{x\to 2}\frac{x^3-x^2-\sqrt{x^2+60}+\sqrt[3]{x^2+60}}{x-2}$

៨-គណនាលីមីត $\lim_{x\to 2}\frac{\sqrt{2+\sqrt{2+\sqrt{2+...+\sqrt{2+x}}}}-2}{x-2}$

៩-គណនាលីមីត $\lim_{x\to 2}\frac{\sqrt{x+\sqrt{x+\sqrt{x+...+\sqrt{x+2}}}}-2}{x-2}$

១០-គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 1}\dfrac{(1-\sqrt{x})(1-\sqrt[3]{x})(1-\sqrt[4]{x})}{(1-x)^3}$
6. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{x+1}+\sqrt[3]{x-1}}{x}$

2. $\lim\limits_{x\to 0}\dfrac{x+1-\sqrt[3]{3x+1}}{x^2}$
7. $\lim\limits_{x\to 1}\dfrac{x^3-1}{2x-\sqrt[3]{2x-1}-\sqrt[4]{x}}$

3. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x-3}-\sqrt[3]{x-1}}{x-2}$
8. $\lim\limits_{x\to 2}\dfrac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

4. $\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x-\sqrt[3]{3x-2}}$
9. $\lim\limits_{x\to 0}\dfrac{x}{1-\sqrt[n]{(1+x)(1+2x)...(1+nx)}}$

5. $\lim\limits_{x\to 0}\dfrac{x^2}{x+1-\sqrt[n]{nx+1}}, n\geq 2$
10. $\lim\limits_{x\to 0}\dfrac{1}{x}(\sqrt[n]{1+x}-\sqrt[n]{1-x})$

១១-គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 1}\dfrac{x+\sqrt{x}+\sqrt[3]{x}-3}{x-1}$
4. $\lim\limits_{x\to 1}\dfrac{(x-1)^2}{x-\sqrt[3]{3x-2}}$

2. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{7x^2+1}-2}{x-1}$
5. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}.\sqrt[3]{1+3x}-1}{x}$

3. $\lim\limits_{x\to 0}\dfrac{(x+1)^n-(nx+1)}{\sqrt[3]{1+x^2}-1}, n\in IN^*$

១២-គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x-3}-\sqrt[3]{x-1}}{x-2}$
6. $\lim\limits_{x\to 0}\dfrac{x}{\sqrt[4]{x+1}+\sqrt[3]{x-1}}$

2. $\lim\limits_{x\to1}\dfrac{x^3-1}{\sqrt[3]{x}-\sqrt[3]{2x-1}}$

3. $\lim\limits_{x\to0}\dfrac{x^2}{x+1-\sqrt[4]{4x+1}}$

4. $\lim\limits_{x\to1}\dfrac{(x-1)^2}{x-\sqrt[3]{3x-2}}$

5. $\lim\limits_{x\to2}\dfrac{x^3-8}{\sqrt[3]{x^2+4}-x}$

7. $\lim\limits_{x\to1}\dfrac{\sqrt{x+1}-\sqrt{x^2+1}+\sqrt{x^3-1}}{\sqrt{x^2-1}+\sqrt{x^3+1}-\sqrt{x^4+1}}$

8. $\lim\limits_{x\to8}\dfrac{\sqrt{x+1}-3}{\sqrt[3]{x}+\sqrt{x-4}-\dfrac{x}{2}}$

9. $\lim\limits_{x\to1}\dfrac{x+\sqrt{x}-2}{\sqrt[3]{x}+\sqrt[4]{x}-2}$

10. $\lim\limits_{x\to2}\dfrac{x^3-\sqrt{x^2+60}}{\sqrt[3]{x-3}+\sqrt[4]{x-1}}$

១៣-ចូរគណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to1}\dfrac{\sqrt{2x-1}+\sqrt{3x+1}-\sqrt{5x^2+4}}{x-1}$

2. $\lim\limits_{x\to1}\dfrac{x+\sqrt[3]{2x-1}-\sqrt[3]{4x^2+4}}{x-1}$

3. $\lim\limits_{x\to0}\dfrac{x^2}{\sqrt[3]{x^2+1}-\sqrt[3]{x^2-1}-\sqrt[3]{x^2+8}}$

4. $\lim\limits_{x\to2}\dfrac{x-\sqrt[6]{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

5. $\lim\limits_{x\to1}\dfrac{\sqrt[3]{4x^2+4}-\sqrt{2x+2}}{x-1}$

១៤-ចូរគណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\cos x}{1-\tan^3 x}$

2. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{3-4\sin^2 x}{2\cos x-1}$

6. $\lim\limits_{x\to0}\dfrac{1-\cos 2x}{\sin^2 x+\sin^3 x}$

7. $\lim\limits_{x\to0}\dfrac{2x-x^2-\sin^2 x}{\cos x-x+1}$

3. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\cos x-\sin 2x}{4\cos^2 x-3}$

4. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\tan^4 x+\cot^4 x-2}{1-2\sin 2x}$

5. $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-8\sin^3 x}{4\cos^2 x-3}$

8. $\lim\limits_{x\to 0}\frac{\cos x-\sqrt{x^2+1}}{x^2+\sin^2 x}$

9. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{1-\sqrt[3]{\tan x}}$

10. $\lim\limits_{x\to 0}\frac{x^4-\sin^4 x}{\sqrt{x^2+1}-\cos x}$

១៥-ចូរគណនាលីមីតខាងក្រោម ៖

1. $\lim\limits_{x\to 2}\frac{\sqrt[3]{2^x+4}-2}{4^x-16}$

2. $\lim\limits_{x\to 1}\frac{2^x-\sqrt{2x+2}}{4^{2x-1}-x^2-2x-1}$

3. $\lim\limits_{x\to 1}\frac{2^x-\sqrt{1+3^x}}{\sqrt{4^x-1}-3^{\frac{x}{2}}}$

4. $\lim\limits_{x\to 0}\frac{e^x-x-1}{e^{2x}-2xe^x+x^2-1}$

5. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{e^{3x}-1}$

6. $\lim\limits_{x\to 1}\frac{x+\sqrt{x}+\sqrt[3]{x}+...+\sqrt[n]{x}-n}{x-1}$

7 . $\lim\limits_{x\to 1}\frac{\sqrt[m]{x}+\sqrt[n]{x}-2}{x-1}$

8 . $\lim\limits_{x\to 1}\frac{(1+\sqrt[m]{x})(1+\sqrt[n]{x})-4}{x-1}$

9 . $\lim\limits_{x\to 1}\frac{(1+x^m)^n(1+x^n)^m-2^{m+n}}{x-1}$

10. $\lim\limits_{x\to 0}\frac{\sqrt[m]{1+x^n}-\sqrt[m]{1-x^n}}{x^n}$

១៦-ចូរគណនាលីមីតខាងក្រោម ៖

1. $\lim\limits_{x\to 1}\frac{x-\sqrt[n+1]{(n+1)x-n}}{(x-1)^2}$

2. $\lim\limits_{x\to 1}\frac{1}{(x-1)^2}\left[x-\sqrt[n]{\frac{nx^{n+1}+1}{n+1}}\right]$

3. $\lim\limits_{x\to 1}\dfrac{\left[x-\sqrt[n+1]{\dfrac{(n+1)x^n-1}{n}}\right]}{(x-1)^2}$ 4. $\lim\limits_{x\to 1}\dfrac{\sqrt[m]{x}-\sqrt[n]{x}}{x-1}$

១៧- ចូរគណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$ 6. $\lim\limits_{x\to 0}\dfrac{2\cos x+\cos 2x-3}{x^2}$

2. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 4x}{x^2}$ 7. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x^3}$

3. $\lim\limits_{x\to 0}\dfrac{1-\cos^3 2x}{\sin^2 4x}$ 8. $\lim\limits_{x\to 0}\dfrac{2\sin x-\sin 2x}{x^3}$

4. $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 3x}{x^2}$ 9. $\lim\limits_{x\to 0}\dfrac{1-3\cos 2x+2\cos 3x}{x^2}$

5. $\lim\limits_{x\to 0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$ 10. $\lim\limits_{x\to 0}\dfrac{1-\cos x\cos 2x\cos 3x\ldots\cos nx}{x^2}$

១៨- គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$ 6. $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos x}}{1-\sqrt{\cos 2x}}$

2. $\lim\limits_{x\to 0}\dfrac{\cos x-\sqrt{\cos 2x}}{x^2}$ 7. $\lim\limits_{x\to 0}\dfrac{1-\cos^n x}{x\sin x}$

3. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2\cos x}-\sqrt{3}}{x^2}$ 8. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3\cos x}-2}{1-\cos^3 2x}$

4. $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{3-\cos 2x}}{x^2}$ 9. $\lim\limits_{x\to 0}\dfrac{1-\sqrt[3]{\cos 2x}}{x^2}$

5. $\lim\limits_{x\to 0}\dfrac{1-\cos(1-\cos x)}{x^4}$ 10. $\lim\limits_{x\to 0}\dfrac{\cos x+\cos 2x+\ldots+\cos nx-n}{x^2}$

១៩- គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 1}\dfrac{\sin \pi x}{1-x^2}$

6. $\lim\limits_{x\to 1}\dfrac{\cos\frac{\pi x}{2}}{1-x^3}$

2. $\lim\limits_{x\to \pi}\dfrac{1-\cos 2x}{(\pi-x)^2}$

7. $\lim\limits_{x\to 1}\left(1-x^2\right).\tan\dfrac{\pi x}{2}$

3. $\lim\limits_{x\to \pi}\dfrac{1-\sin\frac{x}{2}}{(\sin\frac{x}{4}-\cos\frac{x}{4})^2}$

8. $\lim\limits_{x\to 1}\dfrac{1-\sin\frac{\pi x}{2}}{(1-x)^2}$

4. $\lim\limits_{x\to \frac{\pi}{2}}(\pi-2x)\tan x$

9. $\lim\limits_{x\to 2}(4-x^2)\tan\dfrac{\pi}{x}$

5. $\lim\limits_{x\to 2}\dfrac{x^3-8+\tan \pi x}{2-x}$

10. $\lim\limits_{x\to 1}\dfrac{1-x}{\cos\frac{\pi}{x+1}}$

២០-គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{x\to 1}\dfrac{1+\cos \pi x}{(1-x)^2}$

2. $\lim\limits_{x\to 1}\dfrac{1-x}{\sin\frac{\pi}{x}}$

3. $\lim\limits_{x\to 1}\dfrac{1-x^3+\tan \pi x}{1-x}$

4. $\lim\limits_{x\to \pi}\dfrac{\cos\frac{x}{2}}{\pi^2-x^2}$

២១-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim_{x\to 0}\frac{x-\sin x}{x^3}$

2. $\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x}$

3. $\lim_{x\to 0}\frac{1-\cos x\cos 3x}{x^2}$

4. $\lim_{x\to 0}\frac{\sin 2x}{1-\cos\sqrt{x}}$

5. $\lim_{x\to 0}\frac{\tan x-\sin x}{1-\cos(1-\cos\sqrt{x})}$

6. $\lim_{x\to 0}\frac{3\sin x-\sin 3x}{2\sin x-\sin 2x}$

7. $\lim_{x\to 0}\frac{x+\sin x}{\tan x}$

8. $\lim_{x\to 0}\frac{1-\cos(\sin x)}{x\sin x}$

9. $\lim_{x\to 0}\frac{1-\sqrt{\cos 2x}}{x\tan x}$

10. $\lim_{x\to 0}\left[\frac{1}{x^2}\left(\frac{2}{\cos x}-3\cos x+1\right)\right]$

២២-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

2. $\lim_{x\to 0}\frac{\sqrt{x+\cos x}-\sqrt{(1+x)\cos x}}{\sin 3x-3\sin x}$

3. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{x+\cos\sqrt{x}}}{x}$

4. $\lim_{x\to 0}\frac{\sqrt{2+\sqrt{2+2\cos x}}-2}{x^2}$

5. $\lim_{x\to 0}\frac{1+x-\sqrt{1+x^2-2x\cos 2x}}{x^3}$

6. $\lim_{x\to 0}\frac{x+\sin x+\tan x}{x-\sin x-\tan x}$

7. $\lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x}$

8. $\lim_{x\to 0}\frac{\sqrt{1+\cos x}-\sqrt{1+\cos 5x}}{\sqrt{\cos x}-\sqrt{\cos 5x}}$

9. $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{1-\cos 2x\sqrt{\cos x}}$

10. $\lim_{x\to 0}\left[\frac{1}{\sin^2 x}-\frac{2n}{1-\cos^n 2x}\right]$

២៣-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim\limits_{x\to 1}(1-x^2)\tan\dfrac{\pi x}{2}$

2. $\lim\limits_{x\to 1}\dfrac{\tan \pi x}{1-x}$

3. $\lim\limits_{x\to 1}\dfrac{1-\sin\dfrac{\pi x}{2}}{(1-x)^2}$

4. $\lim\limits_{x\to 2}(x^2-x-2)\tan\dfrac{\pi}{x}$

5. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\cos x}{\pi^2-4x^2}$

6. $\lim\limits_{x\to a}\dfrac{\cos x-\cos a}{\sin(x-a)}$

7. $\lim\limits_{x\to \frac{\pi}{4}}\dfrac{1-\sin^3 2x}{\tan x-\cos tx}$

8. $\lim\limits_{x\to 1}\dfrac{\dfrac{\pi}{4}-\arctan x}{1-x}$

9. $\lim\limits_{x\to \frac{1}{\sqrt{2}}}\dfrac{\arcsin x-\arccos x}{1-x\sqrt{2}}$

10. $\lim\limits_{x\to \pi}\dfrac{1+\cos x}{(\pi-x)^2}$

២៤-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim\limits_{x\to 1}(1-x^3)\tan\dfrac{\pi}{1+x}$

2. $\lim\limits_{x\to \pi}\dfrac{\cos\dfrac{x}{4}-\sin\dfrac{x}{4}}{\pi-x}$

3. $\lim\limits_{x\to 1}\dfrac{\cos\dfrac{\pi x}{2}}{1-x^2}$

6. $\lim\limits_{x\to 2}\dfrac{\cot\dfrac{\pi}{x}}{4-x^2}$

7. $\lim\limits_{x\to 1}\dfrac{x^2-1}{\cos\dfrac{\pi}{x+1}}$

8. $\lim\limits_{x\to 1}(1-x)\text{tna}\dfrac{\pi x}{3x-1}$

4. $\lim\limits_{x\to 2}\dfrac{x^3-8+\sin\pi x}{2-x}$

9. $\lim\limits_{x\to 2}\dfrac{1-\sin\frac{\pi}{x}}{x^2-4x+4}$

5. $\lim\limits_{x\to 1}\dfrac{(1-x)^2}{1-\sin\frac{\pi}{x+1}}$

10. $\lim\limits_{x\to -1}\dfrac{\sqrt{\pi}-\sqrt{\arccos x}}{x+1}$

២៥-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim\limits_{x\to\infty} x\sin\dfrac{1}{x}$

2. $\lim\limits_{x\to\infty}\dfrac{2x}{1+x^2}\tan\left(\dfrac{\pi x+4}{2x+3}\right)$

3. $\lim\limits_{x\to\infty}(2x-3)\cos\left(\dfrac{\pi x+2}{2x+1}\right)$

4. $\lim\limits_{x\to\infty}(3x+1)\tan\left(\dfrac{\pi x}{x+1}\right)$

5. $\lim\limits_{x\to\infty}\dfrac{2x+1}{x^2+4}\tan\left(\dfrac{3\pi x+4}{6x-5}\right)$

6. $\lim\limits_{x\to\infty}\dfrac{x+\sin x}{x+\cos x}$

7. $\lim\limits_{x\to\infty}(3x+1)\sin\left(\dfrac{\pi x+1}{x+2}\right)$

8. $\lim\limits_{x\to\infty}\dfrac{x^2}{2x+1}\cot(\dfrac{\pi x+1}{2x-3})$

9. $\lim\limits_{x\to\infty}\dfrac{x^3}{2x^2+1}\cos\left(\dfrac{\pi x+1}{2x+3}\right)$

10. $\lim\limits_{x\to+\infty}\left(\cos\sqrt{x+1}-\cos\sqrt{x-1}\right)$

២៦-គណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim\limits_{x\to\infty}\dfrac{(x+1)^2}{x^2}$

2. $\lim\limits_{x\to\infty}\dfrac{2x+3}{x+\sqrt[3]{x}}$

3. $\lim\limits_{x\to\infty}\dfrac{(2x+3)^3(3x-2)^2}{x^5+5}$

4. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$

២៧-ចូរគណនាលីមីតនៃអនុគមន៍ខាងក្រោម :

1. $\lim\limits_{x\to+\infty}(\sqrt{x^2-3x+7}-\sqrt{x^2-9x+4})$

2. $\lim\limits_{x\to+\infty}(\sqrt{4x^2+3x+1}-\sqrt{4x^2+9x})$

3. $\lim\limits_{x\to+\infty}(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x+\sqrt{x}})$

4. $\lim\limits_{x\to+\infty}(\sqrt{x^2+4x}-\sqrt{x^2-4x})$

5. $\lim\limits_{x\to+\infty}(\sqrt{4x^2+7x+3}+\sqrt{x^2+5x+1}-\sqrt{9x^2+4x+5})$

6. $\lim\limits_{x\to+\infty}\left(2x+3-\sqrt{4x^2-6x+1}\right)$

7. $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+4x^2+1}-x+2)$

8. $\lim\limits_{x\to+\infty}(\sqrt{4x^2+8x+1}-\sqrt[3]{8x^3+12x^2-x+5})$

9. $\lim\limits_{x\to+\infty}(\sqrt[4]{x^4+4x^3}+\sqrt[3]{x^3+3x^2}+\sqrt{x^2+2x}-3x+2)$

២៨-ចូរគណនាលីមីតនៃអនុគមន៍ដូចតទៅ :

1. $\lim\limits_{x\to+\infty}(\sqrt{x^2+3x}+\sqrt{x^2+4x}+\sqrt{x^2+5x}-\sqrt{9x^2-8x})$

2. $\lim\limits_{x\to\infty}(\sqrt[3]{8x^3-4x^2+3x+5}-\sqrt[3]{8x^3+6x^2-2x+1})$

3. $\lim\limits_{x\to\infty}(\ x+4-\sqrt[3]{x^3-6x^2+4}\ )$

4. $\lim\limits_{x\to\infty}(\sqrt[3]{x^3+3x^2}+\sqrt[3]{x^3+6x^2}+\sqrt[3]{x^3+9x^2}-\sqrt{9x^2-4x+3}\ )$

5. $\lim\limits_{x\to+\infty}(\ \sqrt[4]{x^4-8x^3+1}-\sqrt[3]{x^3-9x^2+4x+1})$

7. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x+\sqrt{x}}}$

8. $\lim\limits_{x\to+\infty}(\sqrt{x^2+x}+\sqrt{x^2+2x}+\dots\dots+\sqrt{x^2+nx}-n\sqrt{x^2+8x+1})$

9. $\lim\limits_{x\to+\infty}(\sqrt[3]{x^2+2x+1}+\sqrt[3]{x^2+4x+1}+\dots\dots+\sqrt[3]{x^2+nx+1}-\sqrt[3]{n^3x^3+1})$

10. $\lim\limits_{x\to+\infty}\left[\sqrt[3]{(x+2)(x+4)(x+6)}-x\right]$ ។

២៩-ចូរគណនាលីមីតខាងក្រោម :

1. $\lim\limits_{n\to+\infty}(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}+\dots\dots+\frac{n-1}{n^2})$

2. $\lim\limits_{n\to+\infty}\left[\frac{1+3+5+7+\dots\dots+(2n-1)}{n+1}-\frac{2n+1}{2}\right]$

3. $\lim\limits_{n\to+\infty}\frac{2^{n+1}+3^{n+1}}{2^n+3^n}$

4. $\lim\limits_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots\dots+\frac{1}{2^n}\right)$

5. $\lim\limits_{n\to+\infty}\frac{1^3+2^3+3^3+\dots\dots+n^3}{n^4}$

6. $\lim\limits_{n\to+\infty}\left[\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\dots\dots+\frac{1}{(3n-2)(3n+1)}\right]$

7. $\lim\limits_{n\to+\infty}\left[\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+\dots\dots+\frac{1}{n(n+2)}\right]$

8. $\lim\limits_{n\to+\infty}\left[\frac{1}{2^3-1}+\frac{1}{3^3-1}+\frac{1}{4^3-1}+\dots\dots+\frac{1}{n^3-n}\right]$

9. $\lim\limits_{n\to+\infty}\left(\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\dots\dots+\frac{2n-1}{2^n}\right)$

10. $\lim\limits_{n\to+\infty}\frac{4+44+444+\dots\dots+\underbrace{444\dots\dots444}_{(n)}}{10^n}$

៣០-ចូរគណនាលីមីតខាងក្រោមនេះ

1. $\lim\limits_{n\to+\infty}\left[(1+\frac{1}{2})(1+\frac{1}{4})(1+\frac{1}{16})\ldots\ldots\ldots(1+\frac{1}{2^{2^n}})\right]$

2. $\lim\limits_{n\to+\infty}\left(\cos x.\cos\frac{x}{2}.\cos\frac{x}{2^2}\ldots\ldots\ldots\cos\frac{x}{2^n}\right)$

3. $\lim\limits_{n\to+\infty}\left(\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\ldots\ldots+\frac{2^n}{1+x^{2^n}}\right)$,$|x|<1$

4. $\lim\limits_{n\to+\infty}(1-\frac{1}{n^2})(1-\frac{2}{n^2})(1-\frac{3}{n^2})\ldots\ldots\ldots(1-\frac{n}{n^2})$

5. $\lim\limits_{n\to+\infty}(\frac{2^3+1}{2^3-1}.\frac{3^3+1}{3^3-1}.\frac{4^3+1}{4^3-1}\ldots\ldots\frac{n^3+1}{n^3-1})$

៣១-ចូរគណនាលីមីតខាងក្រោមនេះ

1. $\lim\limits_{n\to+\infty}\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots\ldots\ldots+\sqrt{n}}{n\sqrt{n}}$

2. $\lim\limits_{n\to+\infty}\frac{1}{\sqrt[3]{n}}\left(1+\frac{1}{\sqrt[3]{4}}+\frac{1}{\sqrt[3]{9}}+\ldots\ldots\ldots+\frac{1}{\sqrt[3]{n^2}}\right)$

3. $\lim\limits_{n\to+\infty}\left(\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots\ldots\ldots+\frac{1}{\sqrt{n^2+n}}\right)$

4. $\lim\limits_{n\to+\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots+\frac{1}{n+kn}\right)$

5. $\lim\limits_{n\to+\infty}\frac{1}{n}\left[\cos\frac{a}{n}+\cos\frac{2a}{n}+\cos\frac{3a}{n}+\ldots\ldots\ldots+\cos\frac{(na)}{n}\right]$

៣២-គណនាលីមីតខាងក្រោម :

1. $\lim\limits_{n\to+\infty}\left[\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+.............+\frac{n}{(n+1)!}\right]$

2. $\lim\limits_{n\to+\infty}\frac{1.1!+2.2!+3.3!+...............+n.n!}{(n+1)!}$

3. $\lim\limits_{n\to+\infty}\frac{C_n^0+C_n^1+C_n^2+.........+C_n^n}{1+2+4+8+..........+2^n}$

4. $\lim\limits_{n\to+\infty}\sum\limits_{k=1}^{n}\left(\frac{k^2}{k^4+k^2+1}\right)$

5. $\lim\limits_{n\to+\infty}\frac{\sqrt[n]{n!}}{n}$

៣៣- គេឱ្យអនុគមន៍ $f(x)=\frac{x}{\sqrt{1+x}}$, $x>0$

ក. ចូរស្រាយថា $x-\frac{x^2}{2}\le f(x)\le x$

ខ. គណនា $\lim\limits_{n\to+\infty}\left[f(\frac{1}{n^2})+f(\frac{2}{n^2})+f(\frac{3}{n^2})+.........+f(\frac{n}{n^2})\right]$ ។

៣៤-ក. ចូរស្រាយថា $x-\frac{x^2}{2}\le\ln(1+x)\le x$, $\forall x\ge 0$

ខ. តាង $P_n=\ln\left[(1+\frac{1}{n^3})(1+\frac{4}{n^3})(1+\frac{9}{n^3}).........(1+\frac{n^2}{n^3})\right]$ ។

ចូរគណនាលីមីត $\lim\limits_{n\to+\infty}P_n$ ។

៣៥. គេអោយអាំងតេក្រាល : $I_n = \int_0^{\frac{\pi}{2}} \sin^3 x \cos^{n-1} x$, $n \in IN^*$

ក-គណនា I_n ជាអនុគមន៍នៃ n ។

ខ-គេតាង $S_n = I_1 + I_2 + I_3 + ... + I_n = \sum_{k=1}^{n} (I_k)$

ចូរបង្ហាញថា $S_n = \frac{3}{2} - \frac{2n+3}{(n+1)(n+2)}$, $n \in IN^*$?

គ-គណនាលីមីត $\lim_{n \to +\infty} S_n$ ។

៣៦. គេអោយអាំងតេក្រាល $I_n = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^n x.dx$, $n \in IN$

ក-ស្រាយបញ្ជាក់ថា $(I_n)_{n \in IN}$ ជាស្វ៊ីតចុះ រួចគណនា I_0 និង I_1 ។

ខ-ផ្ទៀងផ្ទាត់ថាចំពោះគ្រប់ $n \geq 2$ គេមាន $I_n = \frac{n-1}{n+2}.I_{n-2}$ ។

គ-គេតាង $P_n = I_n.I_{n-1}$ ដែល $n \geq 1$ ។

គណនា P_n ជាអនុគមន៍នៃ n រួចគណនាលីមីត $\lim_{n \to +\infty} (n^2.P_n)$ ។

ឃ-គេតាង $S_n = P_1 + P_2 + P_3 + ... + P_n = \sum_{k=1}^{n} (P_k)$ ។

បង្ហាញថា $S_n = \frac{\pi}{8} . \frac{n(n+3)}{(n+1)(n+2)}$ រួចទាញរកលីមីត $\lim_{n \to +\infty} S_n$ ។

ង-គណនាផលគុណ $\Pi_n = S_1 \times S_2 \times S_3 \times .... \times S_n$ ជាអនុគមន៍នៃ n ។

ច-គណនា I_{2n} និង I_{2n+1} ជាអនុគមន៍នៃ n ។

៣៧.គេអោយអាំងតេក្រាល $I_n = \frac{1}{2n-1} . \int_0^1 (x^2)^n .dx$, $n \in IN^*$ ។

ក-គណនា I_n ជាអនុគមន៍នៃ n ។

ខ-គណនាផលបូក $S_n = I_1 + I_2 + I_3 + ... + I_n$ ។ រកលីមីត $\lim_{n \to +\infty} S_n$ ។

៣៨.គេអោយអាំងតេក្រាល :

$$I_n=\int_{na}^{(n+1)a}\frac{dx}{\cos^2 x}$$ និង $$J_n=\int_{a}^{(n+1)a}\frac{dx}{\cos^2 x}\ ,n\in IN^*,a>0$$ ។

ក-បង្ហាញថា $I_1+I_2+I_3+....+I_n=J_n$ ។

ខ-គណនា I_n និង J_n ជាអនុគមន៍នៃ n ។

គ-ប្រើលទ្ធផលខាងលើចូរបង្រួមផលបូក :

$$S_n=\frac{1}{\cos a\cos 2a}+\frac{1}{\cos 2a\cos 3a}+........+\frac{1}{\cos(na)\cos(n+1)a}$$ ។

៣៩. គេអោយអាំងតេក្រាល :

$$I_n=\int_0^{\frac{\pi}{2}}e^{-nx}\cos^2 x.dx$$ និង $$J_n=\int_0^{\frac{\pi}{2}}e^{-nx}\sin^2 x.dx\ ,n\in IN$$ ។

ក-គណនា I_n+J_n នឹង I_n-J_n ជាអនុគមន៍នៃ n ។

ខ-ទាញអោយបាននូវតំលៃ I_n នឹង J_n

៤០ . គេអោយស្វ៊ីត $$I_n=\int_{e^{-(n+1)\pi}}^{e^{-n\pi}}\cos(\ln x).dx\ ,n\in IN$$

ក-ស្រាយបញ្ជាក់ $(I_n)_{n\in IN}$ ជាស្វ៊ីតធរណីមាត្រ ។

ខ-សរសេរកន្សោម I_n ជាអនុគមន៍នៃ n ។

គ-គណនាផលបូក $S_n=I_0+I_1+I_2+...+I_n$ រួចទាញរក $\lim_{n\to+\infty}S_n$ ។

៤១. គេអោយអាំងតេក្រាលៈ

$$I_n(t)=\int_0^t\left(\frac{2x+1}{\sqrt{x^2+x}}+\frac{2x+2}{\sqrt{x^2+2x}}+\frac{2x+3}{\sqrt{x^2+3x}}+...+\frac{2x+n}{\sqrt{x^2+nx}}-\frac{2nx}{\sqrt{x^2+1}}\right).dx$$

ក-គណនាកន្សោម $I_n(t)$ ។

ខ-គណនាលីមីតនៃកន្សោម $I_n(t)$ កាលណា $t \to +\infty$ ។

៤២. គេអោយអាំងតេក្រាល $I_n = \int_1^e \frac{x^{-(2n+1)}}{1+x^2}.dx$, $n \in IN, e = 2,71828...$

ក-ចូរគណនាតួ I_0 ។

ខ-ចូរបង្ហាញថា: $I_{n+1} + I_n = \frac{e^{2n+2} - 1}{2(n+1).e^{2n+2}}$ ។

គ-ចូរស្រាយបញ្ជាក់វិសមភាព :

$$\frac{1}{2}x^{-2(n+1)} \le \frac{x^{-2n}}{1+x^2} \le \frac{1}{2}x^{-2n} \quad ,\forall x \ge 1 \text{ ។}$$

ឃ-គណនាលីមីត $\lim_{n \to +\infty} I_n$ និង $\lim_{n \to +\infty} (nI_n)$ ។

៤៣. គណនាអាំងតេក្រាល :

$$I_n = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan^n x} \text{ និង } J_n = \int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot^n x}$$

៤៤. គេអោយស្វ៊ីត:

$$I_n = \int_0^1 \frac{t^n.dx}{1+t^2} \quad \text{ដែល } n \in IN$$

ក-ស្រាយថា I_n ជាស្វ៊ីតចុះ រួចគណនាតួ I_0 និង I_1 ។

ខ-សរសេរទំនាក់ទំនងរវាង I_n និង I_{n+2} ។

គ-ស្រាយបញ្ជាក់ថា $\frac{1}{2(n+1)} \le I_n \le \frac{1}{2(n-1)}$, $\forall n \ge 2$ ។

ឃ-គណនាលីមីត $\lim_{n \to +\infty} I_n$ និង $\lim_{n \to +\infty} nI_n$ ។

៤៥.គេអោយអាំងតេក្រាល $I_n = \int_{e^n}^{e^{n+1}} \frac{\ln x}{x^2}.dx \quad \forall n \in IN, e = 2,71828....$

ក-ចូរបង្ហាយថា $I_n = \left(\frac{n}{e^n} - \frac{n+1}{e^{n+1}}\right) + \left(1 - \frac{1}{e}\right)\left(\frac{1}{e}\right)^n$

ខ-គណនា $\lim_{n \to +\infty} I_n$ ។

គ-គណនាផលបូក : $S_n = \sum_{k=0}^{n}(I_k) = I_0 + I_1 + I_2 + ... + I_n$

៤៦. គេអោយអាំងតេក្រាល $I_n = \lim_{\lambda \to +\infty}\left[\int_1^{\lambda} \frac{1}{(1+x)^2}.\sqrt[n]{\frac{x-1}{x+1}}.dx\right]$ ។

ក-ចូរបង្ហាញថា $I_n = \frac{1}{2}.\frac{n}{n+1}$ រួចទាញរក $\lim_{n \to +\infty} I_n$ ។

ខ-គណនាផលគុណ $P_n = \prod_{k=1}^{n}(I_k) = I_1.I_2.I_3.......I_n$ ជាអនុគមន៍នៃ n ។

៤៧. គេអោយស្វ៊ីត : $I_n = \int_0^e x^n.dx \quad , n \in IN, e = 2.71828...$ ។

ក-គណនា I_n ជាអនុគមន៍នៃ n ។

ខ-គណនា $S_n = \frac{1}{I_0} + \frac{1}{I_1} + \frac{1}{I_2} + ... + \frac{1}{I_n}$ ។

គ-ទាញបញ្ជាក់លីមីត $\lim_{n \to +\infty} S_n$ ។

៤៨. គេអោយស្វ៊ីតចំនួនពិត $(I_n), n \in IN$ ដោយ :

$$I_n = \int_0^{\frac{\pi}{2}} \cos^n x \cos nx.dx$$

ក-គណនាតួ I_0 និង I_1 ។

ខ-ស្រាយថា I_n ជាស្វ៊ីតធរណីមាត្រ រួចគណនា I_n ជាអនុគមន៍នៃ n ។

គ-គណនាផលបូក $S_n = I_0 + I_1 + I_3 + ... + I_n$ រួចគណនា $\lim_{n \to +\infty} S_n$ ។

៤៩.គេអោយស៊ីត

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n .\sqrt{\tan x.dx} \quad , n \in IN$$

ក-គណនាតួ I_0 រួចបង្ហាញថា $I_n \; n \in IN$ ជាស៊ីតចុះ ។

ខ-គណនា$I_n + I_{n+2}$ ជាអនុគមន៍នៃn ។

គ-ស្រាយថាចំពោះគ្រប់ $n \in IN^* : \frac{1}{2n+3} \le I_n \le \frac{1}{2n-1}$

រួចគណនាលីមីត $\lim_{n \to +\infty} I_n$ និង $\lim_{n \to +\infty} nI_n$ ។

៥០ .គេអោយអនុគមន៍:

$$y = f_n(x) = \int_{nx}^{(n+1)} e^{-t^2}.dx \qquad \left(n \in IN \quad e = 2.71828...\right)$$

ក-គណនាដេរីវេ :$y' = f'_n(x)$

ខ-ចំពោះគ្រប់ $n \in IN^*$ គេសន្មត$\Omega_n = f'_n(1)$ ។

ចូរគណនា $S_n = \Omega_1 + \Omega_2 + \Omega_3 + ... + \Omega_n$ រួចទាញរក $\lim_{n \to +\infty} S_n$ ។

៥១.គេអោយអនុគមន៍ f កំណត់លើ $[0, \pi]$ ។

ក-ចូរបង្ហាញថា:$\int_0^{\pi} xf(\sin x).dx = \pi \int_0^{\frac{\pi}{2}} f(\sin x).dx$ ។

ខ-អនុវត្តន៍:គណនា $I = \int_0^{\pi} \frac{x \sin x}{\sqrt{1 + \sin^2 x}}.dx$ ។

៥២.គេអោយអនុគមន៍ពីរ$f(x)$ និង$g(x)$ កំណត់ក្នុង$[a,b]$ ។

ក-ចូរស្រាយបញ្ជាក់អោយឃើញថា:

$$\left|\int_a^b f(x)g(x).dx\right| \le \sqrt{\int_a^b f^2(x).dx \times \int_a^b g^2(x).dx}$$ ។

ខ-អនុវត្តន៏: ចូរស្រាយបញ្ជាក់វិសមភាព

$$\left|\int_0^1 \sqrt{\frac{a\cos x + b\sin x}{1+x^2}}.dx\right| \le \frac{\sqrt{\pi}}{2}\left(a^2+b^2\right)^{\frac{1}{4}} \quad a,b \in IR$$

៥៣-គណនាលីមីត: $\lim\limits_{n\to+\infty}\left[\int_1^{1}\sum_{p=n}^{n+k\pi}\left(\frac{t^{p-1}}{1+t^p}\right).dx\right]$

៥៤-គេអោយស្វ៊ីត: $I_n = \int_0^1 \frac{x^n}{\sqrt{x^2-x+1}}.dx$

ក-គណនាតួ I_0 និង I_1 ។

ខ-រកទំនាក់ទំនងរវាង I_n ,I_{n+1} និង I_{n+2} ។

គ-អនុវត្តន៏: ចូរគណនា $k = \int_0^1 \frac{x^4}{\sqrt{x^2-x+1}}.dx$ ។

៥៥-ដោយប្រើនិយមន័យអាំងតេក្រាលកំនត់ចូរគណនាលីមីត :

ក. $\lim\limits_{n\to+\infty}\left[\frac{1}{n}\left(\tan\frac{a}{n}+\tan\frac{2a}{n}+\tan\frac{3a}{n}+......+\tan\frac{na}{n}\right)\right]$

ខ. $\lim\limits_{n\to+\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+......+\frac{1}{n+n}\right)$

គ. $\lim\limits_{n\to+\infty}\left[\frac{\sqrt[n]{n!}}{n}\right]$ ។

៥៦-គណនាលីមីត

ក/ $\lim\limits_{x\to+\infty}\left(\frac{2x^2+x+3}{2x^2+3x+1}\right)^x$

ខ/ $\lim\limits_{x\to 0}\left(\cos x\cos\frac{x}{2}\cos\frac{x}{4}....\cos\frac{x}{2^n}\right)^{\frac{1}{x^2}}$